UNITED ARAB REPUBLIC

THE INSTITUTE OF NATIONAL PLANNING

Memo. No. 96

<u>Economic Models</u> :
Difinitions & Classification

by

Dr. M. M. El-Imam

27th November 1961

Reprint: June 1966.

1 - INTRODUCTION:

It has become customary, in dealing with problems of economic policy in general, and of planning in particular, to make resort to some type of a model. It is true that many of those who do not favorize the mathematical approach to economic thinking, especially, in the field of policy and planning, would refuse the term "model" to start with. However, some reflection on the whole procedure, would show that-whatever one's way of looking at things is, one should be suing-though by implication-a certain model, albeit an erroneous one !!

To build an explicit model is, in fact synonomous to the construction of a certain theory, perhaps in a formal way. Theorization, though in itself is the basis of the development of science, is in fact an art. And good art would require high talents and not merely the comprehension of a certain kit of tools which would be applied at will in whichever way one likes. Neither the attempt made here nor any attempt offered by some more capable author than the present one, would pretend to provide the reader with the necessary and sufficient means of transforming his own thinking into an "operationally meaningfull" or "mathematically rigorous" framework.

However, every one of us should benefit a lot from studying more carefully the bases and limitations of this relatively new (in use but not in origin) weapon. Let us give an example. When Keynes forwarded his contribution to contemporary economic thinking under the cover of "The General Theory of Employment, Money & Interest", many writers discussed parts of the set of new ideas therein. Some challenged his "propensity to consume"; others his "liquidity preference", and so on. Still some others (and there might be a lot of them up till now !) found the book difficult to read and probably completely beyond their reach.

Then came Hicks with his "Mr. Keynes and the Classics". In that important article, Mr. Hicks reduced the Keynesian system of thinking into a small number of equations, and did the same with respect to what came to be known as the classical doctrine. It is true that Hicks' attempt was greatly facilitated by the emergence of the General theory itself, with all its logical arguments, psychological laws and the like. But Hicks might be considered responsible for the diffusion of the Keynesian thinking in that he provided it in a nutshell. His attempt is not a perfect substitute for the theory. But it is a contribution of some other kind. Reducing the new theory to its bare principles, and doing the same for the set of theories grouped by Keynes under the heading of "classical theory", Hicks succeeded in popularizing both systems of thinking, making short-hand and concrete comparisons, and even suggesting improvements.

To this attempt, which might be considered a landmark in the field of economic model-building, many of the young students of economics owe a lot. It has become customary to recall, as soon as one speaks about the Keynesian theory, that compact diagram summarizing the 45° line of identity, the curve expressing the consumption function (which is, by the way more meaningful than the confusing term "propensity to consume") and the parallel curve which introduces the level of investment, and ends with determining the point of equilibrium. It becomes possible also to think of more complicated systems, and allegedly more "general" theories.

Thus, it is not necessary to start theorizing after going through the same steps suggested by Hicks or by somebody else. What one should invest one's time more profitably into is the following : Given a certain group of theories, problems, and objectives, what are the concrete steps that one should follow in order to get hold of the true kernel of each of them, render them amenable to direct statistical observation, and subject them to historical vertification, or to future projections?

To answer this rather specific question one should start first by defining economic models, discussing their contents, and investigating rules of their classification and specification. With such a background picture one can attempt a more full and systematic answer to the above question. The present paper is confined to the former set of problems, while discussion of techniques of model-building with special reference to econometric investigations and planning problems are considered in another paper, following shortly.

DEFINITION OF A MODEL

The above discussion enables us to visualize the need for, and the meaning of a model. By a model we mean a formalization of a given theory. This formalization usually takes the form of a set of mathematical relationships which express the main economic relations introduced by that theory. Such a definition implies that we have at disposal a given number of economic variates, that these variates are well defined, though they may not be directly observable or measurable, and that we can formulate the contents of the mathematical relationships connecting them, though their form might still remain undetermined. In most cases a direction of casuality is obvious.

In order to understand more fully the meaning of a model model, it is suitable to discuss a bit further the need for having one. In engineering or architecture, it is customary to build an illustrative model for a given project, in order to obtain a fairly good idea of the ins and outs of the project before being finally committed to the suggested design. Such a model stands as an inexpensive experimentation whose need is highly appreciated when the design has not been attempted before.

In economics, and especially in problems of economic policy, experimentation is either impossible or expensive, usually unhumane. It works by abstraction rather than by direct control. One cannot sit satisfied with isolating a number of people, experimenting with e.g. different rates and types of taxation in order to decide which one to fall upon. There is a need for a priori determination of the potential effects of each and deciding upon the most appropriate before getting committed to any. This brings out clearly certain characteristics of models which might be familiar to those acquainted with theoretical economics.

At their bases they are genuine abstractions. They represent idealistic states which would occur given certain conditions. Once abstraction is introduced, a whole multitude of phenomena would follow. In particular the level and limits of abstraction have to be determined, thus leaving room for an infinite number of possible models. Further, not being accompanied by subsequent experimentation, the exact shape of the relationships might be left undetermined. This gives the model a larger scope of generality, but a smaller one of clarity and definiteness. On the other hand, a lot would depend on the use to which the model is put. Sometimes this use reflects a historical phase which brings certain topics to the surface leaving others as unimportant. Thus models of the business cycle which occupied the interest of economic thinkers two or three decades ago, have given way to models of economic growth in current literature.

An interesting feature of the problem would thus be the following. If we are to make use of a given model, we would require to measure its exact, or may be approximate, dimensions in a given case. Thus it might be found true that consumers follow a certain model in deciding the size of their consumption expenditure, e.g., responding in their behaviour to changes in, and only in, their incomes. But the exact degree of their response might differ from one economy to another or even in the same economy as between different points of time.

For this purpose another concept is introduced: that of a <u>structure</u> by which is meant the specific form and values attributed to the characteristics of the model in a given case. It follows that each structure, though numerically different, is capable of being defined through the abstract theoretical model. One possible way of defining a model is hence to consider it as the <u>set of all possible structures</u> which are explainable by means of a given theory.

III - <u>PURPOSES & USES OF MODELS</u>

Since economic theory is subdivided into different departments, one would expect that models can be also used to serve different fields of economic analysis. Thus international trade models deal primarily with problems arising in this one field of economic theory. The types of variables and relationships appearing in such models would clearly be tied up to their purpose.

The most frequently encountered type of model is the so-called "income-determination" model. It has become popularized as a result of the contributions of Keynes and followers. It has been used as a suitable guide to general economic policy in free capitalistic economies. Other models which are used in the field of economic policy are those which are classified as fiscal policy models. These pay much more attention to the variables and parameters under government control, such as government expenditures, tax rates, etc....

Some models concentrate on final demand problems, while others are oriented towards a study of inter-industrial relationships. In Section V we shall have an opportunity to discuss various types of models, and hence their potential uses and limitations.

But apart from these specific uses, one can distinguish three main purposes of models:

1. The first is analytical. By this we mean, analysing certain economic situations, and attempting at giving an interpretation of some historical conditions. This is the type of approach which is encountered in theoretical analyses and applied econometric research. Nothing normative is suggested here. Of course such models can be used as a basis for policy recommendations. But their formulation in the first place is oriented towards problems of interpretation and explanation. From an empirical point of view, these models can be used for the analysis of historical situations.

In fact, we can consider this purpose as served by two distinct groups of models:

a) The first group is "expository". This is usually the simplest type of model, and is frequently used in textbooks. Thus the Keynesian approach is usually illustrated by the simple income-determination model, where investment is considered exogenous, consumption is a function of income, with the familiar income definition equation. This model helps to emphasize the main point of departure of Keynesian thinking, and plays an important part in more serious studies of economic problems.

b) The second group is "descriptive". Here, a more comprehensive model is generally required. Some of the models explained in theoretical studies can play this role. But, generally speaking, this purpose is best served by econometric models.

2) The second purpose is forecasting and projection. The emphasis is changed here from the past and present to the future. By forecasting we mean the estimation of the potential values of the variables explained by the model. This is generally used for the purpose of estimating the future values of such variables. In principle such an operation can be made also for present of past points of time. The problem becomes, concretely speaking, one of estimating certain magnitudes which can be eventually confronted with observations.

It is clear that the proper treatment of forecasting requires the knowledge of the true values of the characteristics of the model. From a theoretical point of view, any analytical model can be always transformed into a form which is suitable for purposes of forecasting even through its characteristics remain numerically unspecified. This point will be explained more fully in another place.

With projection the question is slightly changed. Here one does not merely attempt at knowing the potential values of the variables if the economy is left to itself. Rather one requires to find out such values when some extra conditions are imposed. The conditions would generally take the form of defining certain values, chosen at will, for some of the characteristics of the model. In the terminology of the next section, one specifies certain values for some exogenous variables and/or parameters. One might even determine directly the values of some jointly dependent variables and calculate the values of the remaining ones by means of the model. These uses are quite helpful in problems of policy-making and planning. In fact target setting approach to planning makes an extensive use of projections.

Here the main difference from forecasting is that in forecasting we usually possess some extra information on the values of the characteristics of the model, and solve for the values of the jointly dependent variables. The main use of projections is in the evaluation problem, where new types of policy are to be tested and evaluated. This usually requires the introduction in the model of some relationships which would not have appeared if the purpose was purely descriptive.

3. The third purpose of using models is decisional. There are two general approaches here. The first is to develop some analytical model, then reformulate it in a decisional form, exactly as in dealing with evaluation problems. The main difference between this problem, which might be called the planning problem, and the evaluation problem, is that we start here by defining the objectives to be achieved, and derive from them the values of the policy "parameters" which would conform with the objectives. In the evaluation problem we start by defining the values of the policy parameters, and estimate their effects on the rest of the model.

A more direct approach is to define some function of the variables in the model, called the preference function, which has to be optimized. Besides, there are usually given some boundaries on the values of certain variables. With this amount of information one has to solve the model in such a way that satisfies all conditions simultaneously. The solution is of course a function of the given boundaries and it depends on the function which is to be optimized. The mathematical methods used for this purpose fall under the headings of activity analysis and linear programming.

Thus, whether we are dealing with forecasting, projection or decision-making, we have to start from some well-defined analytical model. Before we turn to the study of the types of such models, let us consider first their contents.

IV - CONTENTS OF ANALYTICAL MODELS:

The contents of the analytical models are the variables included and the relationships connecting them.

a) Variables :

In analytical models one is mainly interested in studying the behaviour of the economic structure or of parts therefrom. The first step in constructing a model is to define the range of the model. This takes the form of defining those economic elements whose values are supposed to be explained by means of the model. These elements are called the endogenous variables. If any other variables are to be included in the model they are to be treated as exogenous, in the sense that they are not to be explained by means of the model, but are given through some extra system, perhaps another model. The important point is that this latter type of elements, though they may be variable, are given as data.

This latter type would include all non-economic variables such as time, natural events, population, etc....... It would also include all those economic variables which, through the process of abstraction or owing to lack of some satisfactory economic interpretation, are considered to lie beyond the scope of the model. Thus the level of prices is taken as data in a model of consumer's behaviour, under competitive conditions. Government expenditure is usually considered exogenous since it does not follow any spontaneous economic law.

It is clear that the specific choice of variables to be considred as endogenous determines the success of the model-builder in explaining facts. Most of the controversy in economic literature concentrates around this problem. Thus Keynes' success was due to the fact that he has chosen the level of income as an endogenous variable and not as exogenous as the classics have done. In the meantime, the treatment of investment in some of his models as exogenous did not greatly affect the value of his findings. On the other hand, he treated the capital stock as exogenous (in fact even constant). Others have taken care of this problem, thus dynamizing his model and extending it to problems of growth, i.e., beyond his major concern of short-run equilibrium.

Another subdivision of variables becomes due when we deal with dynamic models. In these models some endogenous variables (as will as exogenous ones) appear with lags. In the chronological sense, these variables have not to be determined since they are already determined. Therefore both exogenous variables and lagged values of endogenous or exogenous variables are grouped together under the heading: "predetermined" variables. Consequently another group stands alone containing the current values of the endogenous variables. This group is called the "jointly dependent" variables. This terminology arises from the fact that the composition of the model is such that this group contains variables which depend upon each other and upon all or some of the predetermined variables.

b) Types of Equations:

Models differ in their contents, and this arises mainly with respect to the mathematical form in which they are expressed. In the first instant, their contents can be expressed in terms of some implicit functions connecting the variables included. This is usually the form in which most theories are expressed. Economists are generally satisfied with statements such as: Consumption is some function of the level of income and its distribution; investment is a function of the rate of interest and the marginal efficiency of capital, etc... In some cases they might even limit these functions by saying something about their first, and may be also second-order derivatives. Whether such functions are linear or curvilinear, or some other type of function is generally beyond the scope of theorization, at least for a good part of theory. Other relationships are also expressed in terms of inequalities. Such inequalities might involve the variables in some explicit combinations or they might involve some functions of them.

Generally speaking the most familiar form of equations assumes linearity. This is supposed to give a good approximation within certain limits of the values of the variables, since every curve can be expressed in terms of a straight line close to some point. The assumption of linearity might become highly preferable when dealing with large models involving some complicated mathematical procedures, such as the case with linear-programming models. The important point, however, is that one should not lose sight of the fact that linearity is only meant as simplification, and holds only for a limited range by means of approximation. In some cases the assumption of nonlinearity might be necessary e.g., as in the liquidity preference function. Nonlinearity becomes then one of the characteristics of the models, which determine the shape of the model and its results. However, in many cases it might be possible to introduce linearity through expressing the relations in a linear form, not between the original variables but between some functions of them, e.g., their logarithms (as in the case of demand functions with constant elasticities).

In their original form, the relationships included in the model express the shape of some part of the structure of an economy. This is what is called the structural equations, in order to distinguish them from relationships obtained from them by solving two or more equations together for some variable. Thus the consumption function obtained on the basis of the psychological law expressing the behaviour of the consumer is a structural equation. But the multiplier equation obtained by solving this equation together with equation defining income, cannot be considered as structural. Such equations are usually called derived equations, and their validity hinges on the validity of each and all structural equations used for purposes of their derivation.[2] This means that they possess a much less degree of autonomy, than the structural equations.

The structural equations themselves can be subdivided into a number of categories (six categories) as follows:

1. <u>Definitional Equations</u>:

These are equations which are meant to define certain variables in terms of some others in a definitely-known manner. They hold true whatever the values of the variables involved, and they are sometimes considered as <u>identities</u>. Examples are the definition of value as the product of volume into price ; the definition of total consumption as the sum-total of consumption expenditure on different commodities, etc... Thus they may take linear as well as nonlinear forms.

Although such equations should hold exactly true, one might frequently encounter cases where a definitional nonlinear equation is replaced by an approximate linear one. This type of approximation is favorized by Tinbergen especially in his models which include money as well as physical variables. Thus if V is the value at a certain point of time and v is the change in it, and the same is true for Q & q (for quantity) and for P & p (for price), then we have:

$$V = Q \cdot P \qquad (1)$$

and $$(V + v) = (Q + q) \cdot (P + p) = Q.P + Q.p + q.p + q.p$$

or, $$v = Q.p + q.P + q.p$$

Ignoring the products of deviations (q.p.) and substituting for Q & P their average values (observed ones, thus considering deviations as deviations from the averages) we can transform the nonlinear definitional equation (1) holding for the observed values into a linear form:

$$v = \bar{Q}.p + q.\bar{P} \qquad (2)$$

where $\bar{Q}$ & $\bar{P}$ are the observed averages, and :

$$v = V - \bar{V}, \qquad q = Q - \bar{Q}, \qquad p = P - \bar{P}$$

2. Balance Equations :

This type of equations is similar to the above one, in that it is frequently used to define certain variables in terms of the others. However many such equations are not evident by themselves, and are revealed only by means of a coherent accounting system. In many cases such equations are applied as statistical checks on statistical observations. In others they might be the only available way of observation. Examples of this type are the balance of payments, the balance equation between income and expenditutes, etc... Thus in the latter equation we can make sure that income equals the sum of all types of expenditures, each being estimated separately. On the other hand, one can make use of this equation to estimate investment (or private consumption) as a residual. In all cases the relationship holds exactly true by definition, since it is expressed in terms of expost variables.

3. Equilibrium Equations:

In theoretical models, one encounters equations which are very similar to the balance equations. These need not hold always true, especially as they involve exante variables. Thus in market models we have the equilibrium condition that quantities supplied and demanded should be equal. Such equations need not be identically true. They are satisfied only under certain conditions, viz., those of equilibrium. If equilibrium does not exist, they would fall down, and new variables might be introduced to allow for deviations from equilibrium. They can then be transformed into definitioanl equations, defining these deviations.

In static analysis such deviations might be ignored and the model can be assumed to express the status of an equilibrated economy. In dynamic analysis such a procedure might not be always advisable. The important point is that in econometric analysis one might easily slip in the error of treating these equations as identically true. In fact much of the trouble met in econometric investigations is due to the lack of fulfilment of such abstract formulae. It is easy to abstract from the follow-up the process of adjustment necessary for arriving at the point of equilibrium. But this is only true when one assumes static conditions which could eventually lead to any equilibrium at all. Even then, one is not completely justified in one's approach unless one can prove that such an equilibrium is really a stable one. This introduces the notion of stability which is in reality a dynamic concept ($\neq$). However, positions of unstable (or neutral) equilibrium are permitted on the understanding that whenever they are hit by any process they can yield a static solution of the system.

However, even if such abstractions are permissible in pure theory (or in some of its compartments), they do not necessarily represent the process of equilibrium itself. The virtual process would require a full description of the (time-) path which may or may not lead to the final position of equilibrium. In other words, the introduction of such types of equations should be coupled with other types describing the magnitude and the direction of responses to deviations from equilibrium.

($\neq$) It is for this reason that some authors prefer not to use the term equilibrium when dealing with static problems. To avoid any notion of movement, concepts like: "optimal budget", "optimal production policy", etc... are preferred.

Therefore, in dynamic models one has to add to conditions of (static) equilibrium other equations allowing for the definition and interpretation of deviations contained in the model. One has only to mention the Cobwebb Theorem in contrast to the simple static supply-demand model.

4. <u>Institutional equations</u>:

Another type of equations possess a property similar to those of the previous equations, in that they need no particular theorization. They do not represent an economic hypothesis which can be tested by observations or verified by means of facts. Rather they express a certain relationship which should hold (given the exact type of information) by the power of laws other than economic laws.

Thus the cash ratio for bank reserves states a practice determined either by tradition or by law. Tax rates and hence tax proceeds are of a similar nature. But one should always make sure that the definitions used and the estimates made for the variables contained in such equations do really correspond to those virtually used in fulfillment of those institutional relationships.

To illustrate this point, let us for simplicity assume that a flat income-tax rate is given. The tax proceeds would therefore be equal to total incomes times that rate. But to accept this, one must be sure that the estimate used for income is really equal to "income declared", which is not always the case. This might be due to evasion, as well as to differences between concepts of income. (apart from differences as to the period to which income refers). Even worse such differences might follow some sort of a trend (e.g., evasion might grow smaller on time).

Therefore, though such institutional relationships are not subject to random variations due to lack of representativeness of theory to actual behaviour, they are still subject to a great amount of deviations because of the above-mentioned reasons. Econometric formulations of such equations should, hence, take account of them.

5. Technical relationships :

A good part of economic theory deals with production and its organization. Production itself is an engineering process, determined in the first instant according to the available techniques and knowledge of them. No amount of economic reasoning can justify the introduction of a certain type of input in a certain production process. But the choice of the process itself from a number of alternative processes is a typical economic problem, apart from questions of technical feasibilities. Further, the economic theoretician might have some scope for defining the manner in which he can describe the process of production. For example, he can assume that it varies in a continuous or a discontinuous manner. In other words he is at liberty in defining (by way of approximation) the shape of the production surface.

Thus the functions expressing the methods of production used to obtain a certain product or a group of products, have to be determined on a technical rather than on an economic basis. The specific variables included and the shapes of the functions assumed belong to the field of technology. However, the more general the treatment is, or the more aggregative, the more weighty the economic element becomes. The assumptions usually made in this context e.g., the use of the best technology available, the constancy of the state of technology, the similarity in behaviour of different firms, etc... are merely simplifying assumptions, introduced at will by the economist in order to reduce the technological aspect to a minimum. Such concepts as returns and returns to scale, external economies, etc.., are fundamental economic aspects affecting the choice of the type of function to be used.

Again, we have to allow, in econometric formulations for deviations from the specific form chosen for the representation of those technological relationships. A whole multitude of mathematical problems and economic results hinge upon the specific choices made of the forms of technical relationships. Limitations of the model play an important role in this context. Thus the types of production functions chosen in input-output models, though objectionable in many respects, are determined by the nature and limitations of the model itself.

6. <u>Behaviouristic equations:</u>

Here, at last, lies the real contribution of the economist. The ultimate goal of economic theory is to give a fairly acceptable representation of the ways and means used by economic individuals in dealing with each other, and in taking decisions concerning economic aspects of life. These actions and decisions hinge heavily upon the psychological motives and responses and upon a great deal of subjective and objective factors.

To give a coherent picture of real economic life one has therefore to make a number of hypothess concerning the manner in which individuals act and react upon their economic surroundings. This means that certain equations have to be formulated, connecting a selected group of variables and defining their interrelationships in a refutable manner. Here a problem of choice arises, first with respect to the selection of the relevant variables, and second with respect to the type of formula connecting them.

All psychological and non-economic factors have to be introduced in such equations in a definite manner. This is accomplished by means of a set of so-called "parameters". Thus the "marginal propensity to consume" is a certain entity summarizing the sum of the effects of all objective and subjective factors which make the group of consumers react in a definite way to changes in the economic variable: income. The existence of such a propensity is an assumption made by the theoretician which may or may not be true. This assumption implies that income has to be somehow introduced in the model. The constancy of this propensity is another assumption which might break down in a special case. Instead of being constant, it might change in a given way, depending, e.g. , on changes in income distribution, changes in tastes, etc.... Again the theoretician has to introduce a refutable hypothesis as regards the way in which it changes.

In literal economic analysis, such properties of the relationships are only considered in broad terms. Sometimes important properties even escape any discussion at all. This is not the the case with mathematical models, where the shapes of the equations determine in a specific manner the implicit as well as the explicit implications of these hypotheses. On the other hand econometric investigations allow for random deviations from such laws, and play an important role in testing their existence and shape.

The role played by technology in technical relations is taken over here by psychology and other social sciences. Being a social science itself, economics takes a more positive role in formulating behaviouristic laws. However, rather arbitrary limits are drawn between what is considered as economic and what is not.

V. Types of Models:

It is possible to distinguish between different models according to certain criteria:

1. Formulation: this comprises: (a) Approach, (b) Form of equation.
2. Purpose, which has been discussed in Section III above.
3. Coverage: which contains: (a) degree of completeness;
   (b) scope or degree of generality;
   (c) range, or degree of closedness,
   (d) spatial coverage.
4. Type of interdependence.
5. Time: which contains: (a) change specification, or continuity;
   (b) time specification;
   (c) time duration;
   (d) time units.
6. Variables: this contains:
   (a) origin of measurement;
   (b) degree of aggregation;
   (c) variable specification;
   (d) terms of measurement.

These constitute, all in all, 16 criteria, grouped under 6 general headings. As will be shown below the 16 criteria are complementary in the sense that a model can possess one specific type of the subdivisions falling under each of the 16 criteria. This would mean that a complete specification of a model would require a 16-fold description. In the next section we shall build a system of coding which would give a complete specification of any model by means of a 5-letters symbol. Let us now discuss each of the 16 criteria.

1. Approach :

Many authors are satisfied by distinguishing mathematical viz. econometric models. A more careful study shows that one has to distinguish two major types: the deterministic and the stochastic.

i) Deterministic models: By deterministic or functional models we mean those models which assume certain functional relationships between the variables which are to be satisfied exactly without any error. This assumption is clearly beyond any observable empirical case. In fact the deterministic approach is characteristic of theoretical studies. But no theory claims universal applicability. One instance of difference has already been mentioned when we differentiated between models and structures. Another instance falls under what has come to be known as structure-breaks. Here the general form of theory is preserved apart from the fact that certain characteristics procure different shapes at different points. In any case the deterministic approach can take care of such "systematic" differences which are to be distinguished from "erratic" differences which are to be treated by means of stochastic formulations.

One can distinguish two main approaches in formulating deterministic models: the mathematical and the analogue approaches. This distinction refers in fact to the method of solution which is to be applied to the model: the first using methods of algebra and analysis while the second uses engineering constructions, whether hydraulic mechanic or electronic.

The mathematical models can again be subdivided into unspecified and specified models, according to whether the explicit form of the functions is specified or not. The specified type, in its turn, can be <u>abstract</u>, in the sense that the values of its parameters are not explicitly determined, or <u>numerical</u>, with the values of its parameters specified explicitly. The determination of the numerical values can be accomplished through some logical process, i.e., arbitrarily, or according to some statistical evidence, but without any attempt to subject the model itself to direct statistical estimation. Of course this might lead to some contradictions which could have been revealed had there been attempt at direct statistical estimation. Such errors would, however, be detected through experimentation with different sets of values, and discussion of the implications of the model with its numerical values given.

On the other hand <u>the analogue</u> forms are usually used to deal with more complex formulations. The merits of the <u>analogue</u> or engineering approach lie in the fact that it can lead to quick solutions of systems which are either insoluble or prohibitive when treated by ordinary mathematical tools. This implies that this type of model is numerically specified, but generally containing nonlinear and complicated types of equations. Nonlinearities are frequent in economic problems, and it has been recently found imperative to develop some appropriate means of accounting for them in a meaningful manner which would lead to practical solutions. However, little development is shown in this direction, though one might expect more resort to it, especially for problems of policy making.

ii) Stochastic models: These models allow for the unsystematic types of errors or differences between theory and practice.(≠) It has become customary to introduce in the models certain variables which would account for this erratic nature of the behaviour of real life. Making use of conventional statistical theory, these variables are usually specified to possess a stochastic nature, and to follow some appropriate law of random variation. This immediately brings the model into the realm of econometric analysis and the models is called a stochastic or econometric model. Thus instead of speaking about numerical determination of the characteristics of the model, we speak of statistical estimation. In fact the science of "econometrics" has been especially designed in such a manner as to deal with problems of formulation and estimation of this type of models.

A word of warning is due here. The concept "econometric" has not as yet possessed enough stability to allow us to mention it without further reservations. In fact one can sense some change in the line of thinking which is growing stronger all the time. The use made above of the word econometric might be in conformity with what might now be considered as the classical meaning of the word. This meaning is

---

(≠) By systematic we mean here those variations which can be from the economic viewpoint accounted for in a systematic way by some well-specified variables, whether directly measurable or not. However, random variations are themselves capable of changing in time according to some systematic law. In this case we face a problem of auto-or serial correlation which again falls within the domain of statistical and not economic theory. The errors are still considered as stochastic elements.

reconcilable with the requirements of pure theory and the needs of its empirical vertification without any attempt at normative judgements. But recent years have witnessed a growing tendency towards dealing with problems of policy-making and planning. The science has become more normative.

The tendency is best illustrated by the works of Prof. Frisch, the contemporary father of econometrics. During the late twenties and the early thirties, Frisch has been mainly interested in singling out the pecularities of statistical investigations applied to the measurement of economic relationships. Later Prof. Frisch let this stochastic aspect fall to the background, and brought to the fore the problem of the need for more comprehensive models whose parameters are capable of being determined by some extraneous means, and whose value is in solving policy problems. Such models are econometric not in the sense of being capable of stochastical specification and direct econometric estimation, but rather in the sense of being capable of numerical determination and solution of actual questions of economic policy. <u>They are econometric in their use not in their construction</u>. This again is the definition used by Prof. Tinbergen (as conveyed to the author in a private discussion with him).

Strictly speaking, such a definition would bring the model in the range of mathematically specified and numerically determined type mentioned above. Perhaps a major reason for this fundamental change has been the need to introduce models and studies of the Leontief type within the domain of econometric models, which is the nearest possible. Prof. Leontief himself in his "Studies in the Structure of the American Economy" has found that until some fundamental change has taken place in statistical theory so as to bring his huge models within reach of a perfect statistical and stochastical formulation, one has to combine the mathematical-economic approach with elementary and partial empirical studies, perhaps also with extraneous information, obtained, e.g., from technical data or comparable cases.

In fact classical econometricians have become more ready to give way to this general type of compromise. Thus, a prominent econometrician, namely Lawrence Klein has introduced methods of injecting cross-sectional information in aggregative models. Although both types of research are amenable to rigorous econometric estimation, the injection is done in a deterministic rather than a stochastic manner. This approach has been always used freely by Prof. Tinbergen since his outstanding studies of fluctuations in the U.S.A. and the U.K. during the late thirties and early forties. It would be a natural though difficult development of statistical theory to build coherent methods of estimation for such models consuming large numbers of degrees of freedom (in the statistical sense). If this happens then the process of estimation would become econometric in the full sense. We, therefore prefer to keep the term to its classical limits, differentiating all the time between the formulation aspect and the application aspect.

Another new development in the field of model-building is the use of analogue processes in order to bring into existence models which would take care of the stochastic nature of actual life. Of this type we might mention the Monte Carlo models. Again these models are less frequent than those expressed in terms of mathematical equations. Their use has been so far confined to special problems.

Thus the "approach" aspect leads to the following subdivisions:

i- Deterministic: (a) Mathematical: (1) Unspecified
(2) Specified - abstract
(3) Specified - numerical
(b) Analogue - numerical

ii- Stochastic: (a) Econometric
(b) Analogue

2. Form:

The second criterion of models relates to the form used in formulating the equations of the model. It has been mentioned above that in the mathematical approach we might deal with cases where the forms of equations are specified, and others where this form is unspecified. In the former case, the simplest case is the linear. Other forms, which might be labelled, in general, non-linear, include all other types of functional relationships. The need to differentiate the linear type is that it constitutes a special case which is known for its simplicity, both conceptually and mathematically. A good deal of algebra (linear algebra) deals with this special case, and most of the thoroughly studied parts of analysis relate to it. A special case of linearity arises when we assume direct proportionality as is known in many parts of input-output analysis. Such relations are linear ones with zero constant terms. Hence they might be considered as falling under the general heading "linear relationships".

However, the plausibility of the linearity assumption should not hinge only on mathematical considerations. Economic meaningfulness should also have its proper weight. Many objections have been raised against the assumption on economical grounds. However, mathematicians come again to the rescue ascertaining that any function can be approximated within some narrow range by means of a linear one. But the main point is that such a range is always lost sight of in dealing with these models, and soon they are applied without any reservation. It is rather surprising that this range is frequently interpreted as a time-range, although it is within short periods that the most violent fluctuations take place calling for more suitable forms. The treatment of business cycle problems has been frequently a strong example of the failure of the linearity assumption. Many authors have tried to overcome the difficulties by shifting to other types of linear relationships, namely linear difference equations, whose solutions are by no means linear, introducing the minimum amount of nonlinearities in an extraneous manner, (e.g., Hicks' ceiling and floor). Other types of attempting at reviving the linearity assumption are to change the origin of measuring the data. This aspect will be treated under criterion no. (13) below. Still others try to overcome the difficulties by resorting to linear forms in nonlinear transformations of the variables. Thus one might use the logarithms of the variables in place of the variables themselves. There are severe limitations to the exploitation of this outlet, especially when there exist linear definitional relationships between the original variables themselves. On the other hand, attempts at preserving the linear form might easily succeed in the cases where real terms only are used. But in real-financial models, nonlinearities have to be accounted for. Again the changes of origin, together with the introduction of approximations in the definitional relationships, help to overcome the difficulties in a rather crude manner. The econometric models constructed by Prof. Tinbergen are outstanding examples in this connection, (see p. 13 above).

A good deal of economic analysis can be studied without need to specify the form. In this case we can classify the models used under the general heading of "nonlinear" models. Nonlinearity is a general specification in the sense that nonlinear models can contain one or more equations in a linear form, but there should be at least one equation which is nonlinear. The difficulties of nonlinear models might be appreciable when dealing with deterministic mathematical models. But they are more appreciable in econometric investigations, especially since the properties of the statistical estimates in such cases would hold true only under large sample conditions, conditions which are rarely satisfied by econometric investigations.

It is perhaps for this reason that econometricians had turned their faces recently towards engineering systems in order to account in a reasonable manner for nonlinearities. It is perhaps in this direction that modern studies will be directed in the next few years. However, the more we go into this direction the less general the forms of models will be, since they will become at best, sets of a number - which might be large but finite - of analysed structures.

3. Purposes:

This aspect of models has been shown in Sec. III to fall into:

i. Exposition, ii. Description, III. Forecasting
iv. Projection and policy-evaluation, v. Decisional.

The first two types might be considered as analytical models, while the last two can be considered as decisianal in the broad sense.

Some writers would prefer to subdivide this aspect according to compartments of economic theory. However, this latter aspect is taken care of here, by other more fundamental criteria. Emphasis in our categories is put on approaches to problems rather than on their types. As Samuelson has mentioned in the introduction to his "Foundations of Economic Analysis", the application of rigorous mathematical methods to different economic questions has shown that the same system of reasoning appears to be repeated again and again apart from certain conceptual differences. More radical differences would therefore be expected between different approaches rather than between different subjects.

4. Degree of Completeness:

Another criterion for differentiating between models is the degree of completeness. Complete models are those which give an enterpretation of all magnitudes which can be considered as variable in the system. Incomplete models give only a part of the picture. It is clear that every model is, in a sense, incomplete. All would depend on the justification of the choice of those characteristics which are taken as variable and of those which are considered given. The simplest Keynesian model involving income-investment-consumption identity and an income-consumption relation is considered complete with investment given from outside. More elaborate models would consider investment as one of the variables determined within such a system, and the model would become incomplete, if not stretched to cover extra variables including investment.

In fact it is the art of the theoretician to succeed in providing a reasonable short but complete model. The amount of abstraction in such an operation is evident. On the other hand more elaborate econometric methods assume completeness in the models. Mechanical models should also be complete since they usually represent dynamic processes which have to be repreated a large number of times.

5. Scope:

There is another categorization according to the scope covered by the model, which differs from the previous one. Here we are interested in the width of the "market" covered. Thus we can have a model for the market of a specific good or group of goods. Such a model is considered as partial since it would have at its basis a partial (equilibrium) theory. In some cases the model would be confined to a cross-section of the economy, e.g., a sector. ~~Or~~ it may be relating to just one individual in the market or in the sector. Thus we can distringuish three subcategories of partial models:

a) those relating to individuals, e.g., models of consumer's or producer's behaviour.
b) those relating to markets, dealing with the interaction between both the supply and the demand sides of a certain good or factor, or groups thereof.
c) those relating to sectors, which are concerned with summarizing the total activity of groups of individuals belonging to a certain type.

On the other hand, we have the so-called general models, which attempt to explain what they consider to be the key variables of the whole economy. Such models may vary in detail; they might be even incomplete. But nevertheless, the fact that they deal with nationwize relationships, attribute to them a sense of generality which is not possessed by the previous types. It has to be noted that the word "general" is sometimes used in a different sense, to mean the applicability to different cases or levels of national activity, or to be more or less complete. Thus Hicks, after summarizing what Keynes claimed to be "The General Theory", provided what he called a more general theory.

Partial models need not be incomplete in the above-mentioned sense. In fact they are more complete than not. On the other hand, general models might be frequently incomplete. When the sector or market considered by a partial model is a substantial one, the model approaches the borderline of incompleteness. The problem does not raise serious questions so long as we are dealing with mathematical models. But when we come to econometric investigations, it cannot be completely ignored. For this purpose

purpose methods of grafting are used in order to ensure the reliability of the estimates.

6. Range:

This criterion relates also to the degree of coverage, and it expresses the degree of autonomy in the model. The first category is that of closed models. "Closed" here is taken in the formal rather than the more familiar economic sense of having no relations with the outside world. Closed economies might be represented by closed models or by open ones. By closed we mean that all the variables introduced are explained within the model itself , and nothing is left to be determined outside it (except the purely non-economic variables). This property might pose a severe limitation on the model or the modelbuilder, but it is of an important interest in studying dynamic processes. Usually analytical models start by being formulated in a general open manner, but they are reduced to closed ones in the higher stages of analysis.

On the other hand the open models might be open because of either or both of two reasons. The first is that they include some "exogenous" variables which are determined outside the system. The second is that they allow for relations with the outside world which implies that some variable appearing in the model are determined outside the model in the geographical rather than analytical sense. Thus an open model contains two subcategories: that of models with no foreign trade and those with. On the other hand models closed in the present sense are also closed in the economic sense.

7. Spatial coverage:

Apart from the field of international trade, the models known in economic literature are mainly interested in studying the relationships within a certain economy considered as one single entity. They are models of the intra-regional type. On the other hand, models concerned with international trade problems are by necessity models of the inter-regional type. In some versions of theory, models expressing the inter-relations between two countries are used; in others the models are of a multi-country type.

More recently, especially in problems of economic policy and overall planning, some models were introduced which emphasize inter-regional relationships within a given national economy. One-region and multi-region models are gaining more popularity. This approach, though more realistic is in fact much more complicated. By their nature, multi-region models are concerned with inter-regional trade flows. They would thus introduce many variables such as differentials or commodity flows, commodity prices, transport costs, differences in technology, and comparative costs. Thus these models are more valuable when they are of the general, micro, inter-industrial type. This means that much more work has to be done, especially since ordinary economic theory is not oriented towards a study of these groups of problems.

8. Type of Interdependence:

It has been shown before that one can distinguish models which describe the economy as a whole from those which concentrate on one sector or market. These latter, by definition, are of the intra-sectoral type, being concerned with what goes within the sector rather than with relations with other sectors. If we consolidate partial models in one, we end with a general model which concentrates on the main features of the economy regardless of the inter-relationships between the different sectors or markets. This is a process of aggregation and condensation and the resulting model would of course be of the general macro type, and falls within the intra-sectoral category.

However, another approach would be to link rather than to consolidate. In this case new relationships have to be added in order to account for the inter-sectoral relationships, which would then become the central concern of the model. Such inter-sectoral or inter-industrial (if we are mainly interested in the industrial sectors) have recently gained popularity especially for purposes of planning. They are also general models, in that they cover the whole economy. But they stand midway between micro-and macro-models. The point is that in general micro-models we are interested in both inter- and intra-sectoral relationships. This allows such variables as individual prices and quantities to be introduced in the model, thus enlarging its scope. Such was the case with the original (general equilibrium) Wlarassian system.

At present the need for reducing the model to reasonable dimensions has led to a concentration on the problem of interdependence as such. Whether they include intra-sectoral properties, these models are classified as inter-sectoral. Taken together with the criterion no. (14) of variable aggregation, the present categorization can give a rather comprehensive basis for classification.

9. Continuity:

We turn now to a group of criteria relating to time-properties. The first of these defines the manner in which variables are defined as functions of time. Thus the usual manner of measuring variables especially in econometric models, is to consider them as <u>discrete</u> functions of time. This means that for flow-variables we define their magnitudes as flows per finite units of time. Such a definition is implied in static models, since time does not appear explicitly. For dynamic models we can still measure the variables discretely, with the consequence that difference equations become the main tool of analysis. However, if we assume that variables change in a <u>continuous</u> manner with time, differential equations take place, which means that such an approach becomes relevant with respect to dynamic models only. Frequently both the discrete and continuous approaches are mixed together leading to mixed (difference-differential) models. These latter two types are of more theoretical than econometric interest.

10. <u>Time Specification:</u>

One of the oldest and most important criteria relates to the differentiation between static and dynamic methods of analysis. <u>Static</u> analysis has proved to be an old - though not always faithful friend of economists. A model is said to represent a static picture when its solution holds irrespective of time. On the other hand a <u>dynamic</u> model would lead to a solution which defines a time-path for each of its variables, in which the values at any point would depend on previous values or on rates of change.

It is a well-known mathematical fact that any dynamic form in the above sense can be transformed into a static one which would define a potential point of equilibrium. However, it can be easily shown that more than one dynamic form can lead to the same static solution. Hence the passage from a static model to a corresponding dynamic one is not unique. In particular the existence of lags and their actual length has still to be determined. This is one reason that accounts for the difficulty of the department of economic thinking falling under the heading "dynamics" and it is indeed an important sphere where potentialities of the mathematical and econometric ways of thinking are evident.

One important consequence should be mentioned here: To convert an economic problem into a static one for purposes of analysis, is permissible and, in many cases, useful. But to test such a theoretical abstraction against facts is highly unwarranted. This is a difficulty which need not face a mathematical model, but is always present with its econometric counterpart. It is frequently left to the econometrician to carry out the unpleasant task of dynamization. If the data do not prove that a static construction is acceptable, one should not reject it as false. The problem of dynamization has to be studied first. In this case the static form might still be found logically correct though not directly applicable. (*)

Static models are sometimes considered as equilibrium models. This emphasizes the fact that the main objectives of such theoretical frameworks are the determination of the point or set of points where everything becomes really static !! There seems to be some inconsistency in this statement (see above p.14). However, the fact that a static situation is a mental abstraction of a special case in a dynamic world, allows us to justify the concept of equilibrium. What is sought in static analysis is not the determination of the number in which equilibrium is achieved, but rather the determination of the point of equilibrium itself. On the other hand, it is customary in dynamic analysis to speak of an equilibrium

(*) Tintner found ("Econometric", pp. 132-4) that a static supply curve is not sustained by facts. This has not to be considered as implying that such a concept is useless. In dealing with the Keynesian system, Klein pointed out that it has to be dynamized before statistically tested.

path. Dynamics is oriented towards the study of such paths and of deviations therefrom, and their development. In this sense, dynamics is more general than statics.

Another difficulty arises from the fact that any study of an equilibrium has to be subordinated with a study of the stability of that equilibrium. It is clear that stability and instability are dynamic phenomena. However, it is customary in static analysis to study these phenomena, not as processes but as mere possibilities. Again the process of abstraction enables us to concentrate on a part of the problem. It is said that if conditions of equilibrium are disturbed, then the consecutive changes might work in the direction of the original equilibrium away from it, or they may not occur at all. Nothing is said about the duration or the actual mechanism at all. Such questions are left to dynamic analysis.

One might add a word about the concept of comparative statics. It is regarded as the study of a new equilibrium arising from changes in the outside data, and the relation between this equilibrium and the old one. This would mean that a certain amount of "exogenous" factors have to be included in the model. In static analysis these remain constant, and are only changed for purposes of comparative statics. In both cases the model remains static.

II. Time Duration

In dealing with dynamic problems one becomes concerned with the determination of the duration to which the analysis hold. However, no sharp distinction exists in economic analysis between what can be considered as short run and what is truly long run.

Broadly speaking short-run problems relate to deviations from equilibrium or to fluctuations around what can be considered as a systematic time path. The most important of these latter are the cyclical fluctuations. On the other hand, long run problems relate to the dynamic theory of growth.

This categorization might seem satisfactory to those who are mainly interested in emphasizing the basic factors affecting the one or the other: the fluctuation or the growth. It is possible therefore to think of mathematical models which treat either problem and to define the list of variables and relationships which would be expected to be present in each case. However, the problem is not as simple as that. And we believe that authors who attribute to their models the adjectives of being short-run or long-run are not, in most cases, fair to their works.

First one might expect that these models are dynamic. It is true that in statics we speak of "short-run" equilibrium and of "long-run" equilibrium. The difference between the two types lies in the list of elements which are considered variable and those which are taken as, not only given, but also constant. Apart from that there is an exact similarity, whether in the techniques of analysis or in the objectives of study. In neither case we are interested in the process by which equilibrium is achieved. Thus the Keynesian static consumption function is usually regarded as valid only for short-run analysis. For longer runs, other variables are introduced in the equation; some of them are in fact dynamic in their essence.

Another difficulty arises with respect to econometric models. In mathematical models, one can satisfy oneself by putting any amount of assumptions one likes such that the analysis can be concentrated within a limited well-defined field. In econometric models, however, such assumptions need not be satisfied. The econometrician when dealing with such models has to consult time series, which should be - for statistical reasons - sufficiently long. In any period of time, however short, there are short- as well as long-run factors at operation. This means that a model aiming at measuring either effect has to allow for the other one in order to obtain a proper measure.

This problem is not completely solved in mathematical models. A striking example is the contrast between Harrod's model and the Hicks or Samuelson models. The former is supposed to deal with the long run problem of growth. The latters are interested mainly in cyclical fluctuations. However the difference between the two sorts of models lies, not in the types of variables or relations considered, but rather in the lags and the resulting difference equations. In fact, in dynamic models the main distinguishing factor is the shape of the solution.

In the field of forecasting and planning there is a tendency to differentiate between short and long run models. The latter tend to be more aggregative, while the former are generally formulated in terms of first-differences of variables. No clear-cut distinction is made in the literature, apart from the labels added by the authors to their own models.

12. Time units:

The problem of time units plays an important role in economic analysis, though it is still as vague as ever. If one takes the econometric point of view, one is inclined to differentiate between annual and sub-annual models. The main factor considered in this subdivision is seasonality, which is absent in the first case. In the latter, one has two ways open: either to incorporate seasonal variables in the model, or to eliminate the seasonal effects

effects before introducing the economic variables in the equations, thus using deseasonalized variables straightaway. Any decision-besides the decision on the way of accounting for the season - would imply some theory of the inter-relationships between time components. Here we might bring back the discussion of the short- and long-run, or of fluctuation and growth. The afore-mentioned decision has to make an explicit assumption as to the relation between the trend, the cycle and the season: whether they are independent of each other or not. This is a point which is often neglected by many model-builders, especially those interested in studying cyclical behaviour.

If we leave the purely econometric problem aside, one finds that some authors define their time unit as the period sufficient for one complete process of a specified type, e.g., the process of production. Such processes usually take a period smaller than a year. On the other hand in mechanical models such a period is usually taken rather short. Thus we have a third type of sub-annual units namely the non-seasonal. In their formulation they are very similar to the annual models. If the period is not specified, we shall classify the model under the annual group.

13. Origin of Measurement:

The remaining set of properties deal with properties relating to the properties of the variables entering the model. The present one has already been mentioned when speaking about the forms used. The first possibility is to use variables in their absolute magnitudes. This is the most familiar way of expressing most variables. Absolute here does not exclude variables measuring proportions or ratios (e.g. price indices). Rather it means that origin from which the variables are measured is their zero point. This means that all variations are incorporated, whether owing to trends or to fluctuations. This leads to large variations which might render the linearity assumption untenable.

To overcome this difficulty two alternatives are open. The variables might be measured as <u>deviations</u> from some equilibrium path or time trend according to the nature of the study. Alternatively, they might be measured as <u>first-differences</u>. This usually assumes - probably implicitly - an underlying linear system, which has to be amended in order to avoid unwarranted trends in the equations if rewritten in terms of the absolute variables. In other words, constant terms are eliminated. It seems therefore that the assumption of linearity is not avoided in this manner. The real merits of this approach appear to lie in its econometric value in dealing with large positive serial correlations appearing in the residuals of econometric relationships. Some model-builders have even gone as far as stating that first differences are stronger means for forecasting, sometimes taking them as zeros, thus ignoring the existence of any model at all: The discussion of the problem of serial correlation would take us far away from our present study, but it is sufficient to mention here that such an approach raises more statistical difficulties than it solves.

14. <u>Aggregation:</u>

In ordinary economic thinking, micro-analysis is usually confined to the study of the behaviour of the unit-individuals: the consumer, the producer, single commodity markets, etc. On the other hand, <u>macro</u>-analysis is concerned with the study of whole groups of such elements. The passage from the one to the other involves a process of <u>aggregation.</u>

Now, the first stage of aggregation is that of formation of the macro-variables which would correspond to the micro ones. A little reflection shows that this can be achieved in many steps. One step is to aggregate over "persons" according to a certain economic function, e.g., all consumers of durable goods in a given market. Another step is to aggregate over all markets (for the same good) for the whole economy, which is in fact automatically done in the previous step, except in spatial models. A third type of step is to aggregate over all goods of a given kind in order to arrive at aggregates which contain different elements possessing a certain common major characteristic. Thus consumers' expenditure is an aggregate over persons, markets and goods, whose elements have one thing in common, namely that they follow a well defined law of behaviour. It is clear therefore that there does not exist a black-and-white distinction, but rather there is a host of levels of aggregation.

One interesting problem faces the model-builder, especially for purposes of econometric investigation. If one is interested in constructing a macro-model, two ways are open. Either one tries to build the model directly in an aggregated form, or one consults the basic micro-relations and derives from them the appropriate macro-ones. The importance of the problem arises from the need to subject the model to direct observation. In the first approach one can see to it that the macro-relation would contain macro-variables which are capable of direct statistical measurement (albeit ideally). This envolves the problem of aggregation over variables, which was mentioned above.

In the second approach another step of aggregation is envolved, viz., aggregation over the micro-relationships themselves. This means that we would end with a set of variables which depend - in a more or less complicated way - upon the (directly) unobservable characteristics of the micro-relations. The former approach is much easier, and it is the one most commonly used, crude as it is. Many attempts have been made to indicate how close or apart the two approaches are under certain assumptions. But there is no single hard and fast rule according to which one can identify both of them in an unequivocal manner.

15. Variable Specification:

It is customary in economic theory to distinguish flow-variables and stock-variables. The former are measured as flows per unit of time, while the latter are considered as magnitudes standing by at any point of time. In general, the first differences (or derivatives) of stock-variables are well-defined flow variables. But the integral of a flow variable need not always have a meaningful entity as a stock-variable. Some models contain only flow-variables, but the others are the stock-flow type, none being exclusively expressed in terms of stock-variables.

Although the presence of stock-variables is itself a dynamic element, it is not by itself sufficient to a dynamic formulation of the model. Thus the presence of investment in many aggregative models, which means by necessity the presence of capital stock was allowed in many statical studies. Again the static theory of the consumer allows for the treatment of stock-flow commodities. Many parts of monetary theory are treated by means of statics and comparative statics. In other words, the specification of variables is not enough by itself to determine whether a model is static or dynamic. It is the method of analysis which is relevant.

16. Terms of Measurement:

Some fields of economic theory emphasize the monetary side of the economy. Corresponding models would therefore comprise financial monetary and price variables and relations. Others are mainly occupied with the real side of problems. The corresponding models are therefore expressed in real terms. It is the experience in model-building that this type is more easily handled, especially in linear models. Models expressed solely in money terms are not generally much more complicated. If we want to introduce monetary aspects as well as real magnitudes, we soon find that many nonlinearities are introduced, and the size of the model grows very rapidly.

From a historical point of view, models in money terms were the first to be used. They went in line with the price-analysis approach in economics which was the main tool of economic analysis. With the emergence of Keynesian thinking, and the widespread use of the income-analysis approach, the upsurge of economic models which followed, were mostly expressed in real terms.

Although models expressed in real terms seem to be easier to manipulate, the concept of "real terms" raises great difficulties and ambiguities. We have only to recall Keynes' attempt to evade the difficulty through expressing all variables in terms of wage-units. His assertion that there does not exist any well-defined general price level which can be used as a deflator is indicative. Hicks considered this treatment as a "red herring", and justified the so-called general price level through assuming constant terms of trade between the two main sectors: consumption and investment. In attempting to construct his general macro-models, Klein faced the problem, especially when he came to define real incomes, while observing the accounting identities between production, income and expenditure. Some variables had to be estimated as residuals from the accounting identities, though their precise meanings were not well-defined.

The difficulty arises again in planning models. In such a case, concentration on variables in real terms is obvious. But if we are planning with the purpose of raising productivity, reducing costs and improving the technological structure, as in fact is usually the case, we have to take account of these changes in our models. The usual way of expressing variables in real terms by means of evaluating them at constant prices becomes inconsistent with the original assumptions. It follows that one has to use a more elaborate technique, usually introducing many nonlinearities in the problems. The real-financial approach becomes the most suitable for the purpose.

Another difficulty arises with respect to "balance variables". The problem is most clearly illustrated by the blance-of payment evaluation. Expressing every variable in real terms by means of a deflator, and using the underlying identities means that we have to consider the "real" balance of payments as the difference between real exports and real imports. The meaning of such a concept is not at all clear. The balance by its very nature is a balance expressing physical as well as price effects. It should always preserve this nature since it reflects the changes in foreign asset position.

The need to use real-financial relationships becomes urgent whenever problems of choice and allocation are present. Attempts to build decisional models of real-financial (interindustrial as well) type have been met by many difficulties. It is hoped that by using mechanical models, such difficulties can be solved, at least partially.

VI. RULES OF CODING AND CLASSIFICATION:

With the growth of model-building techniques, it has become necessary to classify and group different models according to their major specifications. To the author's knowledge, no such attempt has ever been made, and it is high time to take the problem seriously. Some well-known models are labelled according to the type of problem or the author, e.g., the Cobweb model, the Harrod-Domar model, etc. Prof. Frisch sometimes uses dummy names referring to some central feature of his models, e.g., the Channel model, the Kernel model, the Refi model, etc. But it is necessary for any outsider to know the whole structure before being able to translate such names to full specifications of the model.

Now assuming that the specifications given in Sec. V are comprehensive, one migh think of using them, in some abbreviated manner to codify different models. Such a code is of great help in systemizing discussions of the subject matter of model-building. It will also help in problems of recording models and forming a comprehensive bibliography.

The method suggested below is an attempt to obtain a brief code, which, together with the key to it, would specify completely the type of model. The system is flexible, in the sense that further properties can be introduced as well. For example, the field of theory covered by the model can be added as further property. The form of the model itself (structural, reduced-form, etc.) can be another one. One has only to add a new "digit" as is well-known in coding rules. However, the system might be rigid in the sense that further or different subdivisions of the properties discussed are not easily accounted for. For example, if we want to distinguish in the property of spatial specification a new subdivision containing models of a bi-regional type, as distinguished form the multi-regional it will be practically impossible to adjust the present code in order or to allow for this new category. That was the reason why we have gone through all properties in a rather detailed manner. In any case, experience in coding shows that no code is everlasting, and from time to time (or might be from place to place) changes are found necessary.

The usual systems of coding make use of ordinary numerals, but frequently they use alphabetical letters. The latter approach sometimes is tied to initials. But if free from this condition , it looks more helpful since it allows for 26 rather than 10 subdivisions as is the case with numerals. A code based on letters can be said to extend vertically rather than horizontally. For example, we can use one digit for each of the 16 criteria discussed above. This can also be reduced if all permissible combinations of two or at most three properties are grouped under one digit. In fact an attempt was made on this basis, and it was found that an 8-digit code is necessary in our case. As shown below the alphabetical code suggested contains only five places. Further, to it one can add a two-or (may be three-) digit number by which one orders the models belonging to one class. This will be of great use in problems of classification.

The suggested system of coding is summarized in the following five tables, the order of the table defining the order of the place to which the letters belong. The first table gives letters which would occupy the first place, and so on. There are other possibilities of course, but we have attempted to keep as close as possible to the 6 main groups mentioned at the beginning of Sec. V.

Table (1)

4. Completeness:	Complete		Incomplete	
2. Form of Equ.:	linear	nonlinear	linear	nonlinear
1. Approach				
Deterministic:				
Math. Unspecified	-	A	-	B
Math. Specified	C	D	E	F
Math. Numerical	G	H	I	J
Analogue Numer.	-	K	-	-
Stochastic:				
Econometric:	L	M	N	O
Analogue	P	Q	-	-

Table (2)

3. Purpose:	Expos.	Descrip.	Forecasting	Evaluation	Decisional
5. Scope					
Partial					
Individual	A	B	C	D	E
Market	F	G	H	I	J
Sector	K	L	M	N	O
General	P	Q	R	S	T

Table (3)

8. Interdependence:	Intrasectoral			Intersectoral		
6. Range	Closed	Open	External	Closed	Open	External
7. Spatial						
Single region	-	A	B	-	C	D
Entire Economy	E	F	G	H	I	J
Multi-Regional	K	L	M	N	O	P
Two countries	Q	R	S	T	U	V
Multi-country	W	X	-	Y	Z	-

Table (4)

14. Aggregation			Micro			Macro		
13. Origin (≠)			Abs.	T.C.	F.D.	Abs.	T.C.	F.D.
9. Continuity	10. Time	11. Dur.						
Discontinuous	Static	Short	A	-	-	B	-	-
		Long	C	-	-	D	-	-
	Dynamic	Short	E	F	G	H	I	J
		Long	K	-	-	L	-	-
Continuous	Dynamic	Short	M	N	-	O	P	-
		Long	Q	-	-	R	-	-
Mixed	Dynamic	Short	S	T	U	V	W	X
		Long	Y	-	-	Z	-	-

(≠) Abs. = Absolute, T.C. = Trend corrected, F.D. = First differences

Table (5)

12. Time Unit:		Annual	Seasonal	Deseason.	Nonseason
15. Var. Spec.	16. Terms				
Flow	Money	A	B	C	D
	Real	E	F	G	H
	Real/Money	I	J	K	L
Stock-Flow	Money	M	N	O	P
	Real	Q	R	S	T
	Real/Money	U	V	W	X

Using this system any model can be coded as follows; Consider for example the simple Keynesian model. From table (1), we find that it falls under C, viz., Deterministic, mathematically specified, linear (which is usually assumed) and complere. From Table (2) we have P, viz., General, with the purpose of exposition. From Table (3) we have F, viz., intrasectoral, open, for entire economy. From Table (4) we have B, viz., discontinuous, macro-variables, static, short-term. Finally from Table (5) we have E, viz., Flow, real terms, measured annually. Thus the complete code of the model is :

CPFBE

It is clear that some cells of the tables are empty owing to the fact that the corresponding subdivisions are not concurrent. Similarly if we study the tables together, we observe that some of the letters cannot appear simultaneously in the full code. Nevertheless the total number of possibilities is still very large. A systematic investigation of all available models and of all possibilities would help to throw new lights upon the subject of model-building. It has to be mentioned that the same general model can assume more than one code according to the circumstances in which it is constructed. Thus some writers have attempted to reformulate the Keynesian model in money terms for econometric investigations. In this case it takes the code LQFBA. Thus the different versions of the same model can be coded and classified which helps to bring out the main characteristics of the different versions.

===============

In this paper we have studied the problems of definition, uses, contents and classification of economic models. In a following paper we shall discuss actual steps followed in constructing economic models, and their use for problems of economic policy.