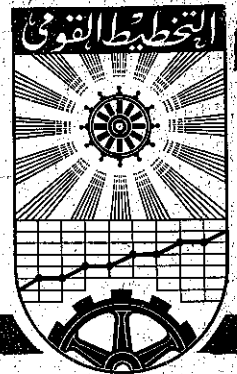


**THE INSTITUTE OF
NATIONAL PLANNING**



Memo. No. 255
Models Used In
Drafting The 20-Years
Plan
(1959 - 1978)

By
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MODELS USED IN DRAFTING THE 20-YEARS PLAN
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Introduction:

During his visit to the National Planning Committee late in February 1960, Prof. J. Tinbergen asked for a formal statement of the mathematical models underlying the computations made in the construction of the national plan. He rightly argued that if such a formulation does not exist in an explicit form one is liable to commit errors and to introduce certain implicit assumptions, without ever being conscious of their presence. The present writer has been charged with the task, and the present paper is an attempt in fulfillment. As will be seen from the following pages, the attempt proves the correctness of the above-mentioned principle. However, it should be borne in mind that we have confined the discussion to the early attempts in building a twenty-year plan, with the predetermined target of doubling national income in twenty years. As is well-known the target has been later modified, and the plan-span has been limited to five years only. The models discussed here served to set the background for the formulation of the five-year plan.

In another paper, the present writer has forwarded certain suggestions as to the principles to be adopted in model-building. Consequently, the theme of discussion adopted here is adapted to those principles, so that future reference can be made with the least possible confusion. In fulfillment of this condition, the discussion of any policy model is subdivided into two main parts. The first is called "structural form", meaning

by that the form of the model which is directly suggested by economic theory. It is the form which serves as a basis for economic analysis of the structure of the economy under discussion. The second part is called the "computational form" or "projectional form". It summarises the actual step adopted in preparing the estimates of national aggregates which would ensue in the plan period. Such computations are not exactly of what one could have called the "decisional form" which is the proper description of a policy model. Further they are not mere forecasts, namely estimates of national aggregates which are most likely to take place during the plan period as a consequence of the natural play of things.

For either form the following information is given in the indicated order:

- (1) Endogenous variables: i.e., variables which are assumed to be explained by the model. Clearly there should be an equal number of equations in the model.
- (2) Data and parameters: By data we mean all other variables which are exogenous to the model. They might be considered as determined by some other model which is logically independent of the model discussed, or through some outside estimates based on extraneous information. Parameters summarize all non-economic factors which determine the behaviour of economic units. As will be seen later, some of the parameters appearing in the computational forms, are not always of the exogenous type. A number of such parameters are determined in such a manner as to ensure the fulfillment of the plan targets. Rigorously speaking such parameters would serve as "instruments" in a properly constructed "policy model". The arbitrary fixation of that group of parameters renders the decisional form as practically irrelevant.

- (3) Accounting frame : It is quite important that the framework of accounts connecting the variables appearing in the model should be explicitly stated. This would bring out clearly the balance relationships among the variables. Further, such a frame would prove as a check against any unwarranted neglect of any relevant variables.
- (4) System of equation: With the above information one can construct the necessary system of equations which would form the required model.
- (5) Solutions of the structural form : It is found convenient to formulate the solutions of the structural form. Two such solutions can be visualized: The reduced form is the one in which all jointly dependent variables, i.e., current values of the endogenous variables, are expressed as functions of predetermined variables, i.e., of data and past values of the endogenous variables. The Separated form, in which each endogenous variable is expressed as a function of exogenous variables and of past values of the endogenous variable itself. These forms are quite helpful in tracing the time paths of the variables, and in expressing the derived rates of growth on the basis of the given parameters. Such a discussion is useful for judging the computational form, which is in a sense some sort of a solution of the underlying model.

As might be expected, the formal model did not exist explicitly in the original papers studied here. It is quite probable that one might choose the wrong form of the structural model. The only guarantee against such a pitfall is the fact that the structural form is indentifiable. It is a well-known fact that if the structural form possesses the merit of being just indentifiable,

there would be a one-to-one correspondence between this form and the reduced form. No. explicit discussion is given here to this problem since it can be shown that the types of models discussed here are in fact indentifiable.

Two approaches are made to the 20-year plan. They are summarized in Memo. No. 71 of the N.P.C. , and Memo. no. 75. The first of these approaches the problem in the mode of two-stage planning recently developed by Prof. J. Tinbergen. However, since Prof. Tinbergen's paper was not available before the above-mentioned attempt was made, the principles indicated therein were not exactly adopted. The second memo. starts with a discussion on the lines suggested by Prof. J. Rudolph. This discussion is then followed by an abridged model on a sectorial basis. This was meant only as an exploration of the possibilities for development, serious discussion of detail being postponed until data became available on developmental projects.

(1)

FIRST APPROACH. MODEL I MODEL I - A.

MACRO-MODEL

The first step was the construction of a macro-model of the Harrod-Domar type. This model helps to determine the magnitudes of national aggregates and their time-path over the 20 years of the plan. We shall first construct the "would-be" underlying structural form of the model, then demonstrate the method by which it was actually applied to the planning problem i.e., the computational form".

I. Endogenous Variables:

- V_t : Gross national product at market prices in year t.
 C_t : Total final consumption expenditure in year t.
 J_t : Gross aggregate investment expenditure in year t.
 O_t : Capital depreciation charges in year t.
 I_t : Net aggregate investment in year t.

Subscript t runs from 0 (for the base year) to e (= 20, for the end-of-plan year).

2. Parameters & Data:

- δ : Ratio of depreciation to national product.
 d_t : Average propensity to consume in year t.
 σ_t : Marginal output/capital ratio (gross to net) in year t.

3. Accounting Frame :

The accounting frame relevant to this model is the simple national income and product account, where there is no distinction of the government and rest-of-world sectors.

4. Structural Form:

The system of equations designed to describe the structure of the economy runs as follow:

$$V_t = C_t + J_t \quad (I.1)$$

$$J_t = I_t + O_t \quad (I.2)$$

$$I_t = (V_t - V_{t-1})/\sigma_t \quad (I.3)$$

$$O_t = \delta V_t \quad (I.4)$$

$$C_t = d_t V_t \quad (I.5)$$

(I) Cf. Memo. 7I, N.P.C. by Drs., E. Sherief, N. Deif, & A.A. Meguid.

Esuations (I.1) & (I.2) are definitional equations derived from the accounting frame. Equations (I.3) and (I.4) are technical equations, the former defining the acceleration principle, while the latter states that depreciation charges are proportional to the degree of utilization of the capital stock as reflected by the level of national product. It might be noticed here that the above formulation of the acceleration principle is the familiar one in theoretical economic literature. It is based on the "behaviouristic" assumption that planned investment is equal to realized investment, and that the current rate of change of product can be safely substituted for the future rate to which current investment pertains. However, in practice one should take account of the maturity period, and thus the rate of change of national product in the R.H.S. of (I.3) should relate to some future period. As will be seen later, the problem of maturity period was not seriously discussed in the practical application and we can therefore retain the above assumption without much loss. Finally there is the behaviouristic consumption function function (I.5) which assumes a variable average (and hence marginal propensity to consume. The variations of the propensity to consume are expressed as functions of t (and not of v), though in practical applications it was implicitly assumed that λ_t is some arbitrary (perhaps discontinuous) decreasing function of V .

It is more convenient to express system (I) in matrix form as follows:

$$\begin{bmatrix} 1 & -1 & . & . & -1 \\ . & 1 & -1 & -1 & . \\ -1/\sigma_t & . & 1 & . & . \\ -\delta & . & . & 1 & . \\ -\lambda_t & . & . & . & 1 \end{bmatrix} \begin{bmatrix} V_t \\ J_t \\ I_t \\ O_t \\ C_t \end{bmatrix} = \begin{bmatrix} . \\ . \\ -1/\sigma_t \\ . \\ . \end{bmatrix} V_t - 1 \quad (2)$$

5. Solution of the Model:

In order to indicate the manner in which this model can be used for practical calculations, it is important to obtain the solution of the system of equations, so that all "jointly dependent" variables can be expressed as functions of "pre-determined" variables.

5.a) Reduced Form:

The first way of solving the model is to transform it into its reduced form, i.e. to express all jointly dependent variables as functions of the single predetermined variable appearing in the structural form, V_{t-1} , and of the structural parameters. First we notice that the inverse of the coefficient matrix of system (2) is:

$$\begin{array}{c|ccccc} I & \sigma_t & \sigma_t & \sigma_t & \sigma_t & \sigma_t \\ \hline \sigma_t(I-\delta-k_t)-1 & \sigma_t\delta+1 & \sigma_t(1-k_t) & \sigma_t(1-k_t) & \sigma_t(I-k_t) & \sigma_t\delta+1 \\ & 1 & 1 & \sigma_t(I-\delta-k_t) & 1 & 1 \\ \sigma_t\delta & \sigma_t\delta & \sigma_t\delta & \sigma_t\delta & \sigma_t(1-k_t)-1 & \sigma_t\delta \\ \sigma_tk_t & \sigma_tk_t & \sigma_tk_t & \sigma_tk_t & \sigma_tk_t & \sigma_t(1-\delta)-1 \end{array} \quad (3)$$

Premultiplying system (2) by this inverse, we obtain:

$$V_t = \frac{I}{1 - \sigma_t(I - \delta - k_t)} V_{t-1} \quad (4.1)$$

$$J_t = \frac{1 - k_t}{1 - \sigma_t(I - \delta - k_t)} V_{t-1} \quad (4.2)$$

$$I_t = \frac{1 - \delta - k_t}{1 - \sigma_t(I - \delta - k_t)} V_{t-1} \quad (4.3)$$

$$O_t = \frac{\delta}{1 - \sigma_t(I - \delta - k_t)} V_{t-1} \quad (4.4)$$

$$C_t = \frac{a_t}{1 - \sigma_t(I - \delta - k_t)} V_{t-1} \quad (4.5)$$

We can reformulate the equations as follows. Let the rate of growth of national product be:

$$\gamma_t = \frac{V_t - V_{t-1}}{V_{t-1}} = \frac{V_t}{V_{t-1}} - 1 \quad (5)$$

It is clear from (4.1) that $\frac{V_t}{V_{t-1}} = \frac{I}{1 - \sigma_t(1 - \delta - \alpha_t)}$. Hence

$$\gamma_t = \frac{\sigma_t(1 - \delta - \alpha_t)}{1 - \sigma_t(1 - \delta - \alpha_t)} \quad (6)$$

Thus system (4) becomes:

$$V_t = (1 + \gamma_t) V_{t-1} \quad (7.1)$$

$$I_t = (1 - \alpha_t) (1 + \gamma_t) V_{t-1} \quad (7.2)$$

$$I_t = (1 - \delta - \alpha_t) (1 + \gamma_t) V_{t-1} \quad (7.3)$$

$$O_t = \delta (1 + \gamma_t) V_{t-1} \quad (7.4)$$

$$G_t = \alpha_t (1 + \gamma_t) V_{t-1} \quad (7.5)$$

Given the values of the parameters, and given the single "initial" value V_0 , we can estimate all subsequent values of V_t . Consequently we can calculate the values of all other variables for all t .

5.b) Separated Form:

It might be desirable to express each variable as a function of the structural parameters and the lagged value of the variable itself. This can be obviously done by solving the structural system which can be shown to be a system of difference equations (with variable coefficients). This might be done by substituting from (1) into (7). However, a more straightforward approach can be made if we assume constant structural coefficients for all t .

is the underlying difference equation. The solution of this first-order difference equation is:

$$x_t = (1 + \gamma)^t \cdot x_0 \quad (11)$$

where x_0 defines the initial value of the relevant variable. It is clear that such initial conditions can also be derived from the original system of equations, given one of them only (V_0 for example). Equations (II) mean that all variables grow at a constant and the same rate all over the 20-years period.

In the general case, where we allow for variable coefficients, equations (10) would look different for different variables. In other words, the different variables will have different rates of growth. One way of expressing this is as follow. Let V_0 be given, then,

$$V_t = \prod_{\tau=0}^{t-1} (1 + \gamma_\tau) \cdot V_0 \quad (12.1)$$

$$J_t = (1 - d_t) \prod_{\tau=0}^{t-1} (1 + \gamma_\tau) \cdot V_0 \quad (13.2)$$

$$I_t = (1 - \delta - \alpha_t) \prod_{\tau=0}^{t-1} (1 + \gamma_\tau) \cdot V_0 \quad (13.3)$$

$$O_t = \delta \prod_{\tau=0}^{t-1} (1 + \gamma_\tau) \cdot V_0 \quad (13.4)$$

$$G_t = \alpha_t \prod_{\tau=0}^{t-1} (1 + \gamma_\tau) \cdot V_0 \quad (13.5)$$

$$0 = \gamma_t - \frac{1}{V_t} \frac{dV_t}{dt}$$

It might be recalled here that γ_t depends on the remaining parameters as shown by (6). This latter equation shows also that we can express σ_t or α_t in a similar manner:

$$\sigma_t = \frac{\gamma_t}{(1 + \gamma_t)(1 - \delta - \alpha_t)} \quad (14)$$

$$\alpha_t = (1 - \delta) - \frac{\gamma_t}{\sigma_t(1 + \gamma_t)} \quad (15)$$

$$\sigma_t = \sigma, \kappa_t = \alpha, \quad \therefore V_t = V, \quad \text{say.}$$

This approach would help in showing the direction and magnitudes by which the structural parameters have to be changed in order to achieve a certain target. We introduce a lag operator, such that, for any variable X, we have:

$$\mathcal{L}X_t = X_{t-1} \quad (8)$$

system (I) can be rewritten follows:

$$\begin{bmatrix} 1 & -1 & . & . & -1 \\ . & 1 & -1 & -1 & . \\ \frac{\mathcal{L}-1}{\sigma} & . & 1 & . & . \\ -\delta & . & . & 1 & . \\ -\alpha & . & . & . & 1 \end{bmatrix} \begin{bmatrix} V_t \\ J_t \\ I_t \\ O_t \\ C_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (9)$$

The determinant of the coefficient matrix is easily found to be:

$$\Delta = (1 - \alpha) - \delta - \frac{1 - \mathcal{L}}{\sigma} = \frac{\sigma(1 - \delta - \alpha) - 1 + \mathcal{L}}{\sigma}. \quad \text{In order}$$

that system (9) of homogeneous equations should have a solution, this determinant should be equal to zero. Since the rate of growth $\mathcal{L}$ is by (6) equal to $\frac{\sigma(1 - \delta - \alpha)}{1 - \sigma(1 - \delta - \alpha)}$. For any variable X (=V, J, I, O, C) we have:

$$\Delta_{x_t} = \frac{\sigma(1 - \delta - \alpha) - 1 + \mathcal{L}}{\sigma} x_t = 0$$

Multiplying both sides by $\frac{\sigma}{\sigma(1 - \delta - \alpha) - 1}$ (provided that denominator is different from zero), we find that:

$$\{1 - (1 + \mathcal{L})\} x_t = 0$$

Hence;

$$x_t = (1 + \mathcal{L}) x_{t-1} \quad (10)$$

It is clear that given three types of parameters, the fourth would follow by necessity. If arbitrary choice is made of any three, the fourth should be evaluated in order to test its reasonability.

6. Target - Setting:

Now, the given target was that national income at year 20 (or V_e , where $e = 20$), should be double its level in the base year (i.e., V_0). Thus:

$$V_e = I' V_0, \quad I' = 2 \quad (16)$$

with a constant rate of growth, this means that:

$$I' = (1 + \gamma)^{20} = 2$$

Hence,

$$\gamma = 3.54\% \quad (17)$$

But if we accept the fact that development is a self-perpetuating process, we should also accept the principle claiming that the aim of planning is the acceleration of the rate of development. In other words, we should allow the economy to gather momentum, and proceed in a process of accelerated growth. Suppose that we subdivide the 20 years into four quinquennial stages. Given that the current annual rate of growth is roughly 0.03, we can safely assume that it is possible to realize the following rates of growth:

First stage	$\gamma = 0.03$	(18)
Second stage	$\gamma = 0.035$	
Third stage	$\gamma = 0.04$	
Fourth stage	$\gamma = 0.05$	

On the other hand, the value of δ can be taken as :

$$\delta = 0.04 \quad (19)$$

Now to the determination of t^* . It seems that what has been virtually adopted was some experimentation with the average propensity to consume, in such a manner that the average output capital ratio (net to net) averages 0,25 for the whole period.

There was no attempt, however, to synchroize the values of this ratio with the values of the other parameters, for every t . this can be seen from the following formulation of the model.

7. Computational Model:

The system of equations which underlies the computations can be expressed as follow:

$$I_t = J_t - O_t \quad (20.1)$$

$$O_t = \delta V_t \quad (20.2)$$

$$J_t = V_t - C_t \quad (20.3)$$

$$C_t = \alpha_t V_t \quad (20.4)$$

$$V_t = (1 + \gamma_t) V_{t-1} \quad (20.5)$$

In matrix form this system is:

$$\begin{bmatrix} 1 & 1 & -1 & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & -\delta \\ \cdot & \cdot & 1 & 1 & -1 \\ \cdot & \cdot & \cdot & 1 & -\alpha_t \\ \cdot & \cdot & \cdot & \cdot & 1 \end{bmatrix} \begin{bmatrix} I_t \\ O_t \\ J_t \\ C_t \\ V_t \end{bmatrix} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ 1 + \gamma_t \end{bmatrix} V_{t-1} \quad (21)$$

Using the inverse coefficient matrix:

$$\begin{bmatrix} 1 & -1 & 1 & -1 & (1 - \delta - \alpha_t) \\ \cdot & 1 & \cdot & \cdot & \delta \\ \cdot & \cdot & \cdot & -1 & (1 - \alpha_t) \\ \cdot & \cdot & \cdot & 1 & \alpha_t \\ \cdot & \cdot & \cdot & \cdot & 1 \end{bmatrix}$$

we can directly obtain the solutions in equations (7).

The calculations are summarized in Table (I), where α_t^* stands for the given marginal propensities to consume, and α_t stands for the correct propensities, as derived from figures given to C_t & V_t . We have shown also the values of σ_t according to (14), and the values of the output/ capital ratios (net to net): $\sigma_t (1-\delta)$.

Table (I) - Calculations for Model I.A.

Year	V_t	C_t	α_t^*	α_t	δ_t	$\frac{I}{\sigma_t}$	$\frac{I}{\sigma_t(1-\delta)}$
0	1000	867	0.87	0.867	(0.03)		
I	1030	893	0.87	0.867	0.03	3.19	3.32
5	1159	987	0.85	0.852	0.03	3.71	3.86
6	1200	1023	0.85	0.852	0.035	3.19	3.32
10	1377	1151	0.83	0.836	0.035	3.67	3.82
11	1432	1165	0.81	0.814	0.04	3.80	3.96
15	1675	1330	0.81	0.794	0.04	4.32	4.50
16	1759	1427	0.81	0.811	0.05	3.13	3.26
20	2138	1660	0.77	0.776	0.05	3.86	4.02

For the whole period, the total increase in gross national product is 1138. The estimated net investment during the same period is 4320, which means a capital/output ratio (gross to net) of 4. The above table shows that no serious consideration was given to the annual capital/output ratios, with the result that they showed unacceptable wide variations.

With $\delta = 0.04$, the capital/output ratio, I/δ can be expressed as a linear function of α_t , for certain level of γ_t . Fig. (1) shows such functions corresponding to the values of γ_t chosen in the calculations. Any arbitrary shift from a certain level of γ_t can

be always accompanied by a suitable shift in γ_t such that σ_t remains at a constant pre-assigned level (e.g. $\frac{1}{\sigma_t} = 4$); or changes in a well-defined manner. This can be determined on the graph by drawing lines like ab or cd, and choosing that line which brings about the desired rates of growth with the least disturbances in the structural parameters.

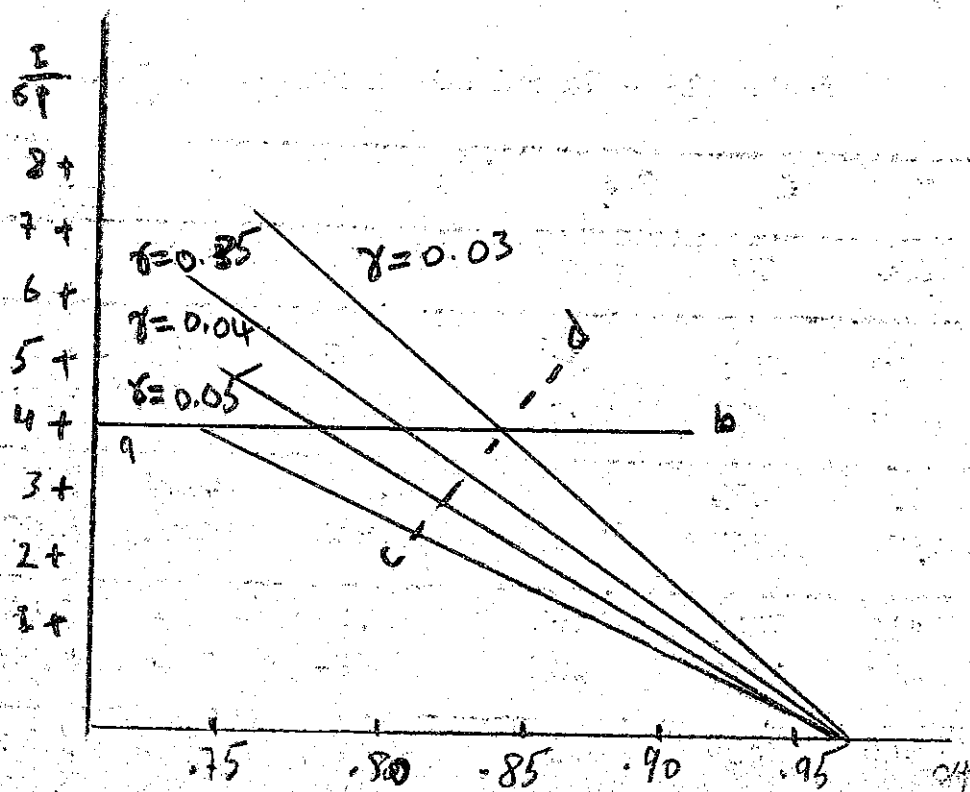


Fig. (I) - Relationships between the capital/output ratio (I/σ_t) & the average propensity to consume corresponding to given growth rates γ_t ($= .03, .035, .04, .05$)

8. Labour Force:

So far we have been solely concerned with the physical side of the problem. Let us now consider the human factor. We assume that total population N_t is given for all t (on the basis of some demographic analysis). The figures given for population were:

$$N_0 = 24.4, \quad N_e = 36.2$$

$$\therefore \Omega = N_e/N_0 = 1.486$$

Putting,

$$(1 + w)^e = 1.02 \dots w = 2\% \quad (22)$$

From (17) & (22) it follows that the average rate of increase of per capita output is 1.15% annually all over the plan period.

Now the ratio of labour force to total population, θ_t ; can be considered constant (e.g., $= \theta$) apart from some change L_t^* in the structure of manpower. Denoting labour force by L , then:

$$L_t = L_t^* + \theta N_t \quad (23)$$

Given:

$$L_0 = 9.3, \quad L_e = 0.70, \quad \theta = 0.40,$$

then,

$$\begin{aligned} L_e &= L_0 + L_e^* + \theta (N_e - N_0) \\ &= 9.3 + 0.7 + 0.4 \times 12.2 = 14.9 \end{aligned} \quad (24)$$

This determines the supply side. The demand side will be taken later in connection with the sectoral model, I.B.

9. Further Comments:

The model discussed in 4 is the familiar Harrod-model. In fact putting $\delta = 0$ (i.e., V is net national income), $(I - \alpha_t) = s$ (the constant average = marginal propensity to save), $I/\sigma_t = c$ (the capital output ratio), and $\gamma_t = g$ (for all t), we obtain Harrod's "warranted rate of growth":

$$g = \frac{s}{c - s} \quad (25)$$

Accepting such a model, we should be aware of its conditions of stability. Since it has been showed by Harrod that any shift from the equilibrium path of growth at a certain moment tends to aggravate, care should be taken that the economy should not be left to wild divergencies. This necessitates a careful choice of the values of the parameters at each point of time.

The appeal of the model seems to originate on the one hand from its simplicity, and from the other from ease (and looseness - and to the mind of the present author, the danger) of indentifying its parameters with decriptive but not analytical concepts, such as the ratio of realised investment (or consumption) to income, the ratio of realized increase in output to investment or to past income. If the structural parameters have any significance at all, the choice of their furture values should not be completely haphazard. It is not sufficient to say that rough estimates of the annual figures are a safe guide for further eleborations based on more detailed models. All will depend on how rough are the estimates. Unless for example, we allow for changes in the degree of utilization of capacity installed, and expicitly justify such changes, the capital/output ratio would cease to be meaningful. Such a term is in fact closely connected with such concepts as the marginal efficiency of capital, in a free (or mixed) society the whole future of the economy will depend on how successful we are in designing the values of the controllable variables or parameters which would bring the private sector into full swing with the theme of the plan. The mere fact that the model includes only some of the most strategic variables calls for a very careful treatment of these variables. It is not sufficient to leave the macro-model at this rather crude state and require the micro-model do the whole trick. The two models should be synchronised together, especially over, time. This will be shown clearly when we discuss the the micro-model.

MODEL I-B. SECTORAL MODEL

Having defined our aggregate variables and estimated their timepaths, the next step is to break them down into sectoral magnitudes. The number of sectors chosen was fifteen as shown in Table (2).

Table (2) - Sectoral capital/output and capital/labour ratios

Sector	capital/output	Capital/labour
	Ratios K^1	Ratios L^1
1. Cotton and agriculture	2.300	500
2. Electricity	3.006	3112
3. Mining and quarrying	2.558	1740
4. Petrol refinement	3.000	1038
5. Basic metallurgy	2.257	1542
6. Basic chemicals	2.719	2476
7. Cement	2.506	1220
8. Other basic industries	2.373	1200
9. Food, drink and tobacco	1.891	435
10. Ginning and pressing	2.992	760
11. Spinning, textiles and clothings	2.260	780
12. Other industries	2.530	1030
13. Construction	5.448	1200
14. Communication and Transportation	3.000	1854
15. Services	1.711	376
Average	2.371	

The ratios indicated in Table (2) will be used later in the process of computations. There will be two sorts of calculations:

First we have certain calculations relating to the end-of-plan year. In principle the underlying model should hold true for any year. Second there are calculations relating to the whole plan period. Again, given the proper model, such calculations would be the sumtotal of the results for individual years based on that model. Finally, both types of calculations should conform with the results obtained through the macro-model discussed in the previous pages. If not, recalculations are due.

A. The Structural Model:

Let us now discuss the model suitable for giving a picture of the whole economic structure at the end-of-year. We denote sectors by superscript k of i ($k=1,2,\dots, k=15$). Subscript t denotes time in years, and as before $t=0$ stands for base year, and $t=e$ for end-of-plan year.

I. Variables: X_t^k : home production of sector k in year t .

M_t^k : imports of goods of the type produced by sector k .

G_t^k : total sources (=total uses) of products of sector k in year t .

I_t^k : deliveries from sector k for intermediate consumption in year t .

C_t^k : deliveries from sector k for final consumption

J_t^k : deliveries from sector k for gross domestic investment in year t .

X_t^{ki} : deliveries from sector k to sector i , on current account.

J_t^{ki} : deliveries from sector k to sector i , on the capital account.

Summations over i will be denoted by the same symbol without superscripts k (or i), while summation over t is denoted by subscript T in place of t .

2. Parameters and Data:

- c^k : autonomic consumption from sector k 's products.
- a^k : marginal propensity to consume products of type k
- g_{ki} : input coefficients from sector k into sector i
- k^{ki} : investment coefficient of product k per unit of increase of i -th sector's output
- μ^{ki} : import coefficients of product k into sector i
- E_t^k : exports from sector k

Aggregate annual consumption, & investment and base year values are given.

- ## 3. Accounting Frame: The accounting frame within which the variables are defined includes first a table of sources and uses. There is a table for each year as follows:

Sec- tor	Sources			Uses				
	Domestic Product.	Imports	Total	Intermed. Consump.	Final Cons.	Domestic Invest.	Ex- ports	Tot.
I								
2								
.								
.								
.								
k	X_t^k	M_t^k	G_t^k	S_t^k	C_t^k	J_t^k	E_t^k	G_t^k
.								
.								
.								
k								
Total	X_t	M_t	G_t	S_t	C_t	J_t	E_t	G_t

B. The Computational Model:

Before discussing the implications of the structural model, or the nature of the conditions imposed by the macro-model, we turn to the set of equations which would best describe the frame of computations adopted in outlining the table of sources and uses at the end-of-plan year.

First it was assumed that given total consumption in that year, and knowing the potential distribution of such an aggregate among sectors as exhibited by statistics of similar economies, we can obtain the breakdown of consumption in year e . This is expressed by equation (33) below, where the α 's denote the average propensities to consume. In the meantime, a breakdown of aggregate investment as obtained from the macro-model, is made but without any indication as to the basis of allocations. Given the sum of consumption and investment demands on each sector, and given the input-coefficients in the base year δ^{ki} one can obtain the corresponding outputs X_e^k , as indicated by equation (34), where δ^{ki} is the kronecker delta. It follows that the difference between this output and the final demand as expressed by consumption and investment defines intermediate consumption. This is shown by equation (35).

At this stage of the analysis, an attempt was made to estimate the excess of the potential increase in demand (as compared with the base year) over the potential increase in total production. This is denoted by M_e^k , and it was obtained through discussions with technicians. In principle, this estimate is not independent of the rest of calculations, since it is based on considerations of maximum output which can be obtained at year e . Using this bit of information we can make our estimate of sectoral imports as in equation (36),.. We are now in a position

to estimate total sources, equation (37). Finally we can obtain the estimates of exports as the difference between total uses (= total sources) and total domestic demand, equation (38).

Thus the system of equations runs as follows:

$$C_e^k = \alpha_e^k C \quad (33)$$

$$X_e^k = \sum_i (\delta_{ki}^k - \xi_{ki}^k)^{-1} (C_e^k + J_e^k) \quad (34)$$

$$\sum_e^k = X_e^k - (C_e^k + J_e^k) \quad (35)$$

$$M_e^k = M_o^k + \bar{M}_e^k \quad (36)$$

$$G_e^k = X_e^k + M_e^k \quad (37)$$

$$E_e^k = G_e^k - (C_e^k + J_e^k + E_e^k) \quad (38)$$

First, C_e is given by the macro-model and we should have (in place of equation 32,) the following relation :

$$\sum \alpha_e^k = 1 \quad (39)$$

This formulation is not quite realistic, since consumption here includes government consumption; and not simply private consumption. If it is true that the α 's are obtained from extraneous information concerning the behaviour of consumers at similar levels of consumption, then this would not take account of the behaviour of the public sector. If we adopt instead formulation (30), we can take account of this fact in the autonomous part C^k , provided that we can obtain estimates of government consumption by sector from some outside sources.

As mentioned before, there was no indication as to the basis of allocation of investment among sectors. In principle one might be justified in taking any arbitrary distribution (and

even size) of domestic investment in year e . However, since size was given by the macro-model, there should logically exist some relationship between investments in year e , and the trend of production in years $e + 1$, $e + 2$, etc... This is expressed by (31), which does not have any counterpart in the computational system. It follows from (34) that the estimate of domestic production would be the production required to cover the final domestic demand only, and not total final demand. The two need not coincide unless we assume a zero balance of payments in each sector. The same applies to intermediate consumption derived from the above calculations. In fact if we substitute from (35) into (38) and compare the result with (37) we find that we should have :

$$M_e^k = E_e^k \quad (40)$$

However, the figures given in Memo. 7I do not satisfy (40), and no reason was given for that.

Let us now consider the investment side of the problem. Given the capital-production ratios k^k we can apply them to the expected increases in the levels of production which define the production targets. Thus the distribution of aggregate investment all over the 20 years, namely $J_t^k = \sum_{t=1}^e J_t^k$; would be:

$$J_t^k = k^k (X_e^k - X_0^k) \quad (41)$$

Total investment requirements is the same as that calculated from the aggregative model, equation (7.2) :

$$J_T = \sum_t J_t = \sum_k J_T^k = \sum_t \left\{ (1 - \alpha_t) (1 + \gamma_t) \right\} \cdot V_{t-1} \quad (42)$$

Hence,

$$\sum_k k^k (X_e^k - X_0^k) = \sum_t \left\{ \prod_{s=1}^t (1 - \alpha_s) (1 + \gamma_s) \right\} \cdot V_0$$

Putting, $\kappa_e = \sum_k h^k X_e^k / X_e$, $\kappa_o = \sum_k h^k X_o^k / X_o$

&

$$\phi_t = \prod_s^t (1 - d_s) (1 + \gamma_s)$$

we can rewrite the last relation as follows:

$$\kappa_e X_e - \kappa_o X_o = \sum_t \phi_t \cdot v_o$$

Since $X_e = \Gamma X_o$ we have the following relationships:

$$(\kappa_e \Gamma - \kappa_o) = \sum_t \phi_t g_o \quad (43)$$

where g is the ratio of value added to global production. Since the κ^k are given, and so are the magnitudes κ_o , ϕ_t and Γ , we can rewrite (43) as follows:

$$\kappa_e = \frac{\sum_t \phi_t g_o}{\Gamma}$$

In other words,

$$\sum_k h^k X_e^k = \frac{1}{\Gamma} \times (g_o X_e ; \sum_t \phi_t) \quad (44)$$

This imposes a restriction on the X_e^k which was nowhere taken explicitly into consideration. In simple words, the calculation of investment in the aggregative model I-A implies an overall capital-output ratio which should conform with the overall capital-production ratio implied by the sectoral model. Otherwise there is no guarantee that the increases in capacities required to meet a certain prescribed level of final demand would involve the same amount of total investment required to realize a certain developmental path.

It seems that a lot of guesswork and iterations were involved in the actual calculation. It is of the utmost importance to know the exact method of iteration, since one should

always be clear with regard to the new set of assumptions, and probably new relations, which are introduced either implicitly or explicitly in each round of the iterative process. If this is not true, subjective biases are liable to creep into the picture, thus leading to untenable results. It is clear of course that one could have adopted the iterative process provided one is about the set up of iterations. Alternatively, one could have attempted directly at substituting the sectoral model straightway in the aggregative model. This might give a direct set of solutions, but the amount of computational work would increase to a large extent. This might be a factor in favour of the iteration is not the same as rough estimation, since by iteration one is always converging towards an "exact" solution to any desired degree of approximation, and not merely attempting at being content with rough estimates based on personal and a noncomprehensive scheme of analysis. The problem of synchronizing two models of the types discussed before will be considered in a later paper.

The Problem of Employment:

Estimates of labour force were given before, (equation 24). With regard to the demand side we first notice that the level of employment L^P in the base year is estimated to be $L_0^P = 7.1$, which means that unemployment in the same year is $L_0^U = 2.2$. Now, given the capital labour ratios ξ^k in the K sectors, and given the sectorial net investments I_T^K all over the twenty years, we can obtain estimates of the increases in sectorial employments through the plan. The estimates can be summarized as follows: Denoting the three sectors, agriculture, industry and services by the three superscripts, a, b & c respectively, then

$$\begin{array}{llll} L^a = 1.7, & L^b = 2.4, & L^c = 2.2, & L^p = 6.3 \\ C^a = 500 & C^b = 920 & C^c = 552 & C^p = 680 \end{array}$$

Thus, total employment increases by 6.3 so that it becomes: $L_e^p = 13.4$. This means that unemployment by the end of the twenty years will be reduced to $L^u = 1.5$, i.e., it will fall to 10% rather than the current 23%.

This concludes our discussion of the first set of models, and we turn now to a discussion of some alternative formulation of the 20 - years plan.

MODEL II (I)

I. Testing Target Feasibility:

As mentioned before, a fixed target was politically determined, and that was the doubling of national income within 20 years. This alternative approach seeks first to test whether such a target is feasible or not. Instead of constructing an overall rate of growth and breaking it down to its components as was done in Model I-A, an attempt was made to construct rates of growth for the main sectors of the economy. The underlying idea is that different sectors usually exhibit different potentialities of growth, and possess different limitations. On the other hand, if it is agreed that the present state of the economy is an undesirable one, then this is due to what one might call "structural disequilibrium". Thus a breakdown to the three sectors: the primary (agriculture), the secondary (industry) and the tertiary (services) would help to spot out the nature of disequilibrium, and to rationalise any rate of growth on the basis of directives designed towards the adjustment of the distribution of activity among those sectors.

We start with the information about population:

$$N_0 = 23.8, \quad N_e = 35.6$$

As in (22), the annual rate of growth of population is w , and the ratio of population at the end of the plan period is

$$\Omega = (1 + w)^e = 1.5 \quad (45)$$

On the other hand, the labour force is estimated at: $L_0 = 7.0$. If we assume that it will grow at the same rate as population, we find that: $L_e = 1.5 \times 7.0 = 10.5$. In other words,

$$\Lambda = L_e / L_0 \quad \Lambda = \Omega = 1.5 \quad (46)$$

We denote the three sectors by the superscripts $k = a, b, c$, respectively. The following ratios are introduced:

$$\rho_t^k = x_t^k / L_t^k \quad \gamma_t^k = v_t^k / L_t^k \quad (47)$$

(I) Cf. Memo. No. 76, N.P.C., by Mr. M. Ibrahim.

They denote average production and output per worker in each sector. The data for the base year are summarized as follows:

Levels of Employment & Productivity in the base year

Sector	K	L_o^k	β_o^k	η_o^k
Agriculture	a	4.0	111.5	74.5
Industry	b	0.7	938.6	303.7
Services	c	2.3	246.0	214.8

If we assume that the distribution of labour force in year e will be the same as in year o, and that the productivities will remain constant, it follows that aggregate income will increase by the same rate as labour force, and hence as population, to the effect that per capita income will remain constant. Thus instead of doubling national income, we will have an increase of only 50%. This might be considered as setting an absolute minimum below which national income should not fall. In the meantime it follows that any attempt at raising national income above that limit means synonemously raising productivity per worker. Hence, we can approach the problem through a discussion of the possibilities of increasing productivities.

Let us denote the ratio of end-of-plan to current production by Φ . Then:

$$x_e^k = \Phi^k x_o^k \quad (48)$$

Let the annual rate of growth of productivity be r , so that we have :

$$\beta_e^k = r^k \cdot \beta_o^k \quad r^k = (1 + r^k)^e \quad (49)$$

On the other hand defining ratios Λ^k similar to that defined by (46) for each sector, it follows that:

$$X_e^k = L_e^k \cdot \beta_e^k = (\Lambda^k r^k)(L_o^k \cdot \beta_o^k) = (\Lambda^k r^k) \cdot X_o^k$$

Hence,

$$\phi^k = \Lambda^k r^k \quad (50)$$

We denote the distribution of the labour force by θ ,

$$\theta_t^k = L_t^k / L_t \quad (51)$$

Hence,

$$\Lambda^k = L_e^k / L_o^k = (\theta_e^k \cdot L_e) / (\theta_o^k \cdot L_e) = \Lambda \cdot (\theta_e^k / \theta_o^k) \quad (52)$$

Further, let:

$$x_t^k = X_t^k / X_t \quad (53)$$

It follows that, $X_o^k = x_o^k \cdot X_o$. Substituting this in (48) we find that:

$$X_e^k = (\phi^k \cdot x_o^k) \cdot X_o \quad (54)$$

Putting $X_e = \phi \cdot X_o$, and summing (54) over k to obtain:

$$X_e = (\sum \phi^k \cdot x_o^k) \cdot X_o$$

we find that:

$$\phi = \sum \phi^k \cdot x_o^k \quad (55)$$

It follows that:

$$x_e^k = \frac{X_e^k}{X_e} = \frac{\phi^k \cdot x_o^k}{\phi} \quad (56)$$

We can rewrite (54) as follows:

$$X_e^k = \left(\frac{\Lambda^k r^k \cdot \theta_e^k \cdot x_o^k}{\phi \theta^k} \right) \cdot X_o \quad (57)$$

Thus, we could break down the growth rates of each sector and of the global production as a whole into their principal elements. However, we are primarily interested in the growth of national income. We shall assume that the ratio of value added to global production in each sector remains constant all over the plan period. It follows that we can define for each sector:

$$\psi^k = v_o^k / v_o^k \quad (58)$$

and,

$$\psi^k = \phi^k \quad (59)$$

Analogous to (53), we have:

$$v_t^k = v_t^k / v_t \quad (60)$$

The overall ratio ψ will then be:

$$\psi = \sum \psi^k \cdot v_o^k \quad (61)$$

Although (59) holds for the individual sectors, a similar relation need not hold for the whole economy unless the ratios of value added to production are equal, which is not true. Finally, we can establish equations similar to (56) and (57), with x replaced by v and ϕ by ψ .

The details of calculations are given in the following table:

Determination of sectorial growth rates

	Sector			Global	National
	a	b	c	Production	Income
r^k	0.02	0.01	0	(0.02)	(0.014)
r^k	1.5	1.22	1.00	(1.494)	(1.30)
Δ	1.5	1.5	1.5	1.5	1.5
θ_e^k	0.53	0.17	0.30	1.00	1.00
θ_o^k	0.57	0.10	0.33	1.00	1.00
Δ^k	1.395	2.550	1.364	1.5	1.5
$\phi^k = \psi^k$	2.09	3.11	1.36	(2.24)	(1.95)
x_o^k	0.267	0.394	0.339	1.000	-
$\phi \cdot x_o^k$	0.558	1.225	0.461	2.224	-
x_e^k	0.249	0.546	0.205	1.000	-
x_o^k	0.30	0.21	0.49	-	1.00
$\psi \cdot v_o^k$	0.627	0.653	0.666	-	1.946
v_e^k	0.32	0.34	0.34	-	1.00

The first step in the calculations was to estimate the possible rates of growth of productivity in each sector on the basis of the current conditions and of the experiences of other countries. Then an attempt was made to adjust the distribution of the labour force in year e , by choosing some appropriate values for θ_e^k . With respect to agriculture the estimate was made on the basis of the condition that the man / land ratio should remain unchanged. With an expected increase in acreage of about 40%, a similar increase in L^a is assumed. On the other hand the services sector was assumed to exert a slight decline in its share since it is a sector which contains a lot of disguised unemployment and it is expected that this sort of unemployment would fall down over the plan period. This leaves the remaining labour force available for industry. Hence we can apply (52) and consequently (50) which determine the growth rates of the sectors. It might be observed here that the assumption of the absorption of some of the disguised unemployment in sector c is contradictory to the assumption of a constant labour productivity in that sector. On the other hand if it is assumed that the cultivated area will increase by 40%, then it need not follow that the resulting increase in production would be also 40% since the added area is expected to be less fertile, at least in the beginning. Indeed, it was assumed in the memo. (75) that this new area will be equivalent to 25% only of the original area, but this assumption did not appear later in the calculations.

Dividing the global productions of the sectors by the national global production, we obtain the ratios defined in (53). Similar ratios are also calculated for sectoral incomes, as in (60). These ratios are then multiplied by the rates of growth of corresponding sectors, and the sumtotal defines the proportionate

increase in national production or income. Thus aggregate global production would increase by 124% while national income increases by 95% only. In other words, on the previous assumptions it seems feasible that national income could be doubled in 20 years provided the required adjustments are realized. However, this would require an increase in global production by more than proportionate as a result of the more than proportionate increase in sector b whose ratio of production to value added is higher than that of the other sectors. This fact is illustrated by the estimates of x_e^k given by (56) and the corresponding ratios v^k .

Dividing the resulting estimates ϕ & ψ by Δ we obtain the ratio of per worker production and income at the end of the plan period to those of the base year:

$$r^p = \phi / \Delta = 1.494, \quad r^v = \psi / \Delta = 1.30 \quad (62)$$

These ratios imply an annual rate of increase for production per capita of 2% , and of per capita income of 1.4%.

2. Investment Requirements:

Having determined the possibilities of growth of each of the three main sectors, we can estimate the investment requirements using the capital/labour ratios. This would entail certain income/capital coefficients. Thus :

$$J_T^k = \xi^k (L_e^k - L_o^k) \quad (63)$$

The corresponding income/capital coefficients are:

$$\sigma^k = (v_e^k - v_o^k) / I_T^k$$

For agriculture the ratio ξ^a is obtained on the basis of estimating the total costs of the high dam (excluding the costs of electrification), and the costs of increasing labour productivity.

This gives $\xi^a = \text{L.E. } 350$ per worker. For industry, it is recognized that the new industries would be of the basic industries type, thus $\xi^b = 1500$. For services, the estimate is put at two-thirds of that of industry, $\xi^c = 1000$. These estimates lead to a total of about L.E. 3000. m. for net investment over the whole period. The corresponding income/capital coefficients are:

$$\sigma^a = 0.579 \quad \sigma^b = 0.275 \quad \sigma^c = 0.215 \quad \sigma = 0.315.$$

3. Estimates of Final & Intermediate Demand:

The next step is to consider the demand side of the problem. For this purpose, estimates of the final demand in year e were made, though no indication is given as to the method adopted. It seems however, that some sort of a Harrodean model was used. Thus it is estimated that the rate of investment will increase from 10.3% in the base year, to 14.8% in year e. This increase is required to meet the acceleration of the rate of growth of income. Consequently we have either to assume an increasing rate v^V , whose average was found to be 1.4%, or to assume a variable capital coefficient. We have only remember here the discussion made with respect to model I-A.

A preliminary estimate of imports in year e puts them at the level of 366 millions. To allow for repayment of foreign loans incurred a deficit in the/balance of payments of 15 m. is expected in the same year. Using the income-definition equation we can obtain the estimate of aggregate consumption in year e. Thus we have the following system of national magnitudes:

$$V_e = \text{given (from previous calculation)} \quad (65.1)$$

$$B_e = \text{given (balance of payments = 15)} \quad (65.2)$$

$$M_e = \text{given} \quad (65.3)$$

$$E_e = M_e + B_e \quad (65.4)$$

$$J_e = \xi_e \cdot V_e \quad (\xi_e \text{ determined a priori}) \quad (65.5)$$

$$C_e = V_e - J_e - B_e \quad (65.6)$$

$$X_e = G_e - M_e \quad (65.7)$$

$$G_e = \sum_k \sum_i (\xi_{ki} - \xi_{ki}^{-1}) (C_e^k + J_e^k + B_e^k) \quad (65.8)$$

The first six equations are the representation of the above arguments. Equation (65.7) expresses the fact that domestic production is the difference between total sources and imports. Now to obtain total sources (or rather total uses), a breakdown of components of final demand was made as among 16 sectors. Using the inverse of the technical matrix an estimate is made of direct and indirect requirements is made. This means that we should have an estimate of the sectorial final demands.

For consumption we have a breakdown first with respect to private and public consumption then among sectors:

$$G_e = C_e^h + C_e^g \quad (66)$$

It is possible to obtain a breakdown of household consumption C_e^h and public consumption C_e^g among the 16 sectors in the base year. Further dividing these figures by total population we obtain per capita consumption. Using Engel coefficients, per capita consumptions in year e were obtained on the basis that household consumption would increase by 20%, while government consumption would increase by 22% which is the corresponding rate of increase of household demand on services. Inflating the resulting figures by the total number of population in year e, the national aggregates were obtained. Clearly some adjustment has to be made, since there is a restriction on the weighted of the consumption

expenditure-elasticities, namely that the sum of expenditure elasticities weighted by the base year expenditures should add up to total expenditure. The use of Engel coefficients would give a higher total for year e than the original aggregate. This is all the more true if we introduce government consumption expenditure in the same scheme. No indication is given as to the process of adjustment adopted.

As in Model I-B, estimates of J_e^k were given without any justification for the specific breakdown adopted. Further, no indication is found as to the principles adopted in obtaining the breakdown of exports among the 16 sectors in year e . In any case, we have a picture of total final demand on the 16 sectors. With this information the inverse of the technical coefficient matrix is applied to final demands as shown by equation (65.8). It is stated that this would give us the total requirements, direct and indirect, from each sector, which have to be met either by domestic production or from importation, i.e., the required total sources G_e^k . Summation over k , gives aggregate sources. Using (65.7) we can obtain total domestic production in year e .

However, if we accept equations (65.8), we would be including in G all the indirect requirements of products whether produced at home or imported. In spite of this fact, the estimated G_e is 3412, as compared with X_e given on the basis of the expected rates of growth for the three main sectors, which amounted to 3745. On the other hand, if we apply (65.7) given that $M_e = 366$, we find that $X_e = 3046$, i.e., $= 1.825 X_0$. Again, this model suffers from lack of synchronization between the sides of production and demand. It is interesting to notice here that the estimates of G_e were compared with base year levels of production in order to determine what has been called "production

targets". In doing that a new set of figures were introduced for base year domestic production. No reason was given for the differences from the original set X_0 , nor for the rather low level of this new set. One reason, of course, might be that these levels are estimated at producers' prices. But then they would not be comparable with the figures of total requirements, which were obtained on the basis of final demand estimated at market prices. If we do accept the assumption of using the technical matrix based on producers' prices, this means that we would be underestimating intermediate consumption. This might be an explanation of the observed differences. The following table summarises the values X_0^k given in this part of the ana-

lysis, together with the original X_0^k and the original values of value added V_0^k . We indicate also the value of X_e^k obtained before, and G_e^k derived from (65.8). The sumtotal of final demand is also given.

k	X_0^k	X_0^k	V_0^k	X_e^k	G_e^k	$(C + J + E)_e^k$
a	446	340	298	932	670	424
b	567	447	213	2043	1484	1034
c	566	524	494	770	1258	862
Total	1669	1311	1005	3745	3412	2320

The outstanding feature of the above figures is that total requirements from sectors a & b are far below the previous estimates of production, while the reverse is true for services, sector c. If one argues that imports would bear the burden of corrections, it is by no means clear how this can be done, especially with respect to services. In other words, unless we assume a drastic change in the ratio of value added to global

production, there is no way of reconciling the two sets of figures. In fact, we can illustrate the inconsistencies of the figures by calculating the intermediate consumptions observed in the base year, and required to meet final demand in year e(I)

sector	a	b	c	Total
o	148	444	72	664
e	48	817	592	1457

The position of sector b, which was assumed before to contribute more to development is quite striking. If the previous figures are accepted, the income assumed to have originated from a certain level of global production would be incapable of sustaining that production at all. Alternatively, if the previous levels of production were feasible, then a higher level of national income would be possible. Neither of the two possibilities was discussed. The problem as a matter of fact was left at that point on the assumption that if a certain level of income was proved feasible, we can go ahead to plan it without any discussion of the implication of the assumption.

A final, though not less important remark, has to be mentioned here. The feasibility test was made on the basis of certain assumptions as regards possibilities of raising labour productivity. If we assume a fixed wage rate, then the labour input would be reduced, with the result that the secondary inputs receive a larger weight. This would call for a change in the technical - coefficients matrix. On the other hand, if we assume that the wage rates would change in such a manner that the increases in productivities are exhausted by appropriate wage rate increase, the result would be a change of the labour input which would compensate the changes in productivities.

(1) Cf. Memo. no. 82, N.P.C., by Mr. M Ibrahim.

But this would mean variations in sectorial wages which had to be somehow taken into consideration, and their effects on equilibrium of the economic system. In any case, there are grave doubts as to the workability of the base year technical-coefficients

4. Further Considerations :

In a later occasion, (1) the model considered here was corrected for some of the observations made above. First, total labour force was allowed to increase by 4.2 instead of 3.5 thus allowing for the change of the structure of the labour force mentioned in model I-A. This increase was distributed between industry (0.2) and services (0.5). Secondly, the fact that the new cultivated area is not equivalent to the existing area was accounted for by taking the rate of increase of productivity in agriculture as 1.5% in place of 2%. The capital/labour ratios $\bar{c}$ were reduced by 100 in both sectors b & c. The resulting income/capital ratios $\bar{r}$ became: 0.455, 0.290, and 0.238 in the three sectors respectively, and 0.300 for the whole economy. The ratio of global production to value added in agriculture was raised to account for the increased requirements when methods of intensive cultivation are used, But no account was made for this factor when later use was made of the technical-matrix. Vectors of final demand remained the same, except for some slight increase in household consumption. But the estimates were higher than previously calculated. This helped to eliminate a lot of the discrepancy, although there still remains a gap, after allowing for imports. The following table shows the original estimates and the new ones (denoted by asterisks). Where $\bar{F}$ stands for final demand.

k	G_e	G_e^*	V_e	V_e^*	F_e	F_e^*	ζ_e	ζ_e^*
a	670	860	622	553	424	429	48	307
b	1484	1729	667	741	1034	1049	817	988
c	1258	1276	666	773	862	873	592	503
Total	3412	3865	1955	2067	2320	2351	1457	1798

As before the figures given here for ξ intermediate consumption including imports.

Thus, no change in methodology was introduced. The corrections are only made for the calculations. Apart from the corrections of the technical matrix which are introduced nothing was made to ensure the consistency of the demand and production sides. All previous comments hold true here also.

Secondly, we have the input-output table:

Delivering Sectors	Receiving Sectors						Total Deliveries
	I	2		i		k	
I							
2							
.							
.							
.							
k							
.							
.							
.							
k							

$$x_t^{ki} = \sum_i a_{ki} x_t^i \quad g_t^k$$

A similar table can be constructed for deliveries on the capital account, with the general element equal to

$$j_t^{ki} = a_{ki} x_t^i$$

Finally, a third table is the import matrix with the general element

$$u_{ki} \quad g_t^i$$

With this information we can build our system of equations.

4. The Structural System:

Given the above accounting frame, we can construct our structural system taking into consideration the accounting relationships between the variables involved. This system would run as follows:

$$G_t^k = X_t^k + M_t^k \quad (26)$$

$$M_t^k = \sum_i u_{ki} G_t^i \quad (27)$$

$$X_t^k = Z_t^k + C_t^k + J_t^k + E_t^k - M_t^k \quad (28)$$

$$Z_t^k = \sum_i z_{ki}^k X_t^i \quad (29)$$

$$C_t^k = \sum_i a_{ki}^k C_t^i \quad (30)$$

$$J_t^k = \frac{1}{2} \sum_i E_{ki}^k (X_{t+2}^i - X_t^i) \quad (31)$$

Equation (26) defines the total of sources (=uses) of the products of each sector. Equation (27) is an import function for each sector. Equation (28) is again a definitional equation of the domestic product derived from the sources and uses table. Equation (29) determines the intermediate consumption of the products of the k-th type, as given by the input-output table. Equation (30) is a consumption function of products of type k. In this equation, C_t stands for aggregate consumption i.e., $C_t = \sum C_t^k$. This implies the following conditions:

$$\sum_k \bar{c}^k = 0, \quad \sum_k a^k = I \quad (32)$$

Finally, equation (31) determines the investment requirements from the k-th sector. In this formulation we have assumed a uniform lag of two years, with equi-proportional distribution. It might be added here that one can also add two definitional equations for X_t^{ki} & J_t^{ki} which are in fact substituted in the R.H.S. of (29) & (31).

If this model has to conform with the findings obtained from the previous model, some extra conditions have to be added. Thus X_t , C_t and J_t are given. Further, we should also have a zero balance of payments, i.e., $M_t = E_t$. It is assumed that E_t^k can be considered as given from extraneous information, outside the above model.